

Lecture Notes on Performance vs. Robustness of a Cantilever

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Source material: Yakov Ben-Haim, 2005, Info-gap Decision Theory For Engineering Design. Or: Why ‘Good’ is Preferable to ‘Best’, in *Engineering Design Reliability Handbook*, Edited by E. Nikolaidis, D. Ghiocel and Surendra Singhal, CRC Press.

A Note to the Student: These lecture notes are not a substitute for the thorough study of books. These notes are no more than an aid in following the lectures.

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¶ Overview:

- Optimal design of cantilever with uncertain load.
- Performance-optimal design
has no robustness to uncertainty: Unreliable.
- Trade-off: Performance vs. robustness.
- Moral: Feasible designs are sub-optimal.
- Moral: Designers are “maximizers”, “optimizers”.

The question is:

Max-optimizers of what?

- Robustness and safety factors.

1 PERFORMANCE OPTIMIZATION

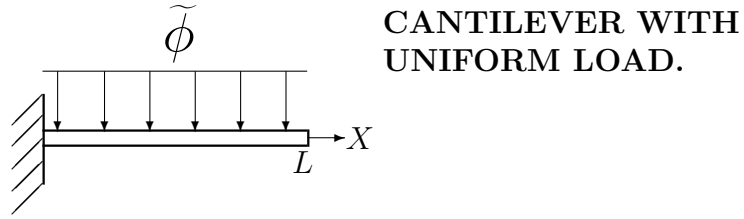


Figure 1: CANTILEVER WITH UNIFORM LOAD.

¶ PARAMETERS:

L = BEAM LENGTH (FIXED).

W = BEAM WIDTH (UNIFORM, FIXED).

ρ = BEAM DENSITY (UNIFORM, FIXED).

$\tilde{\phi}$ = UNIFORM CONTINUOUS LOAD DENSITY (FIXED).

$\sigma_T(X)$ = MAX BENDING STRESS IN SECTION AT X .

$T(X)$ = BEAM THICKNESS (HEIGHT) = **DESIGN FUNCTION**.

¶ PERFORMANCE CRITERIA:

MASS:
$$\min_{T(X)>0} \int_0^L T(X) dX$$

STRESS:
$$\min_{T(X)>0} \max_{0 \leq X \leq L} |\sigma_T(X)|$$

¶ THESE CRITERIA CONFLICT. TRADE-OFF NEEDED.

¶ CONSIDER BEAMS OF MASS θ :

$$\int_0^L T(X) dX = \theta$$

¶ LINEAR-TAPER PROFILE MINIMIZES the MAXIMUM STRESS:

$$\hat{T}_\theta(X) = \frac{2\theta(L-X)}{L^2}$$

¶ RESULTING MIN-MAX BENDING STRESS:

$$R(\hat{T}_\theta) = \frac{3\tilde{\phi}L^4}{4W\theta^2}$$

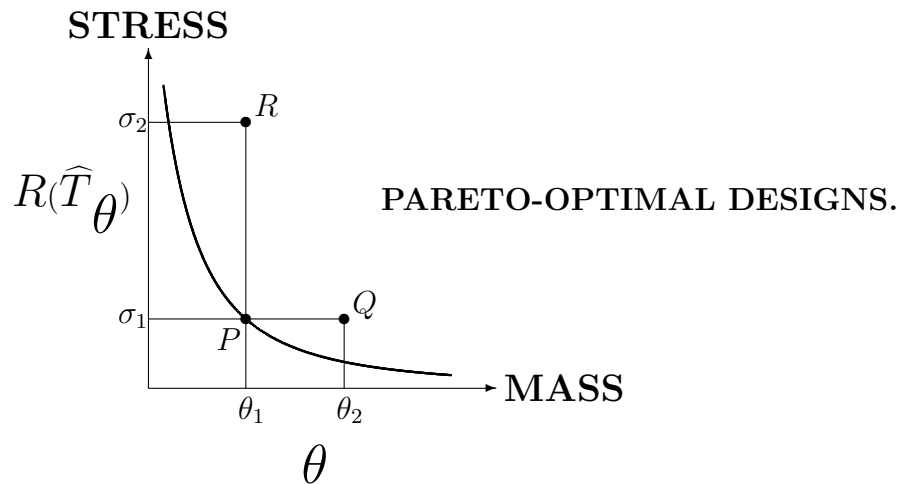


Figure 2: PARETO-OPTIMAL DESIGNS.

¶ DESIGNS ON THE CURVE ARE **PARETO EFFICIENT**:

- PARETO EFFICIENCY is a SHORT-BLANKET IDEA:
PULL UP YOUR BLANKET TO WARM YOUR NOSE,
YOUR TOES GET COLD!
- STRESS CAN NOT BE REDUCED W/O INCREASING MASS.
- MASS CAN NOT BE REDUCED W/O INCREASING STRESS.

¶ DESIGNS ABOVE THE CURVE ARE **SUB-OPTIMAL**.

¶ ARE THE OPTIMAL DESIGNS

FEASIBLE? RELIABLE?

ROBUST TO UNCERTAINTY?

NOPE.

2 ROBUSTNESS TO UNCERTAIN LOAD

$\tilde{\phi}(X)$ = NOMINAL,
 TYPICAL,
 DESIGN-BASE,
 BEST-ESTIMATE
 OF THE LOAD-DENSITY PROFILE.

$\tilde{\phi}(X)$ IS UNDOUBTEDLY WRONG.

$\phi(X)$ = ACTUAL, UNKNOWN LOAD-DENSITY PROFILE.

DISPARITY BETWEEN $\tilde{\phi}(X)$ AND $\phi(X)$ IS AN INFO-GAP.
 INFORMATION-GAP:

DISPARITY BETWEEN WHAT IS KNOWN
 AND WHAT NEEDS TO BE KNOWN
 FOR A GOOD DECISION.

¶ MANY INFO-GAP MODELS OF UNCERTAINTY.

¶ WE WILL USE:

$$\mathcal{U}(h, \tilde{\phi}) = \left\{ \phi(X) : \left| \phi(X) - \tilde{\phi}(X) \right| \leq h \right\}, \quad h \geq 0$$

¶ INFO-GAP MODEL: FAMILY OF NESTED SETS OF EVENTS:

$$h = 0 \quad \implies \quad \mathcal{U}(h, \tilde{\phi}) = \left\{ \tilde{\phi} \right\}$$

$$h \leq h^\bullet \quad \implies \quad \mathcal{U}(h, \tilde{\phi}) \subseteq \mathcal{U}(h^\bullet, \tilde{\phi})$$

h = HORIZON OF UNCERTAINTY.

TWO LEVELS OF UNCERTAINTY:

- UNKNOWN LOAD AT HORIZON h .
- UNKNOWN HORIZON OF UNCERTAINTY: NO WORST CASE.

¶ ROBUSTNESS, $\hat{h}(T, \sigma_C)$, OF DESIGN $T(X)$:

- MAX HORIZON OF UNCERTAINTY WITHOUT FAILURE.
- HOW MUCH CAN WE ERR IN $\phi(X)$
W/O JEOPARDIZING THE BEAM?

$$\hat{h}(T, \sigma_C) = \max \left\{ h : \left(\max_{\phi \in \mathcal{U}(h, \tilde{\phi})} \max_{0 \leq X \leq L} |\sigma_T(X, \phi)| \right) \leq \sigma_C \right\}$$

¶ INTERPRETING THE ROBUSTNESS FUNCTION:

- σ_C : MAX ACCEPTABLE BENDING STRESS.
: DESIGN SPECIFICATION.
: PERFORMANCE-ASPIRATION.
- $T(X)$: DESIGN.
- $\hat{h}(T, \sigma_C)$: ROBUSTNESS TO UNCERTAINTY.
of DESIGN $T(X)$ with ASPIRATION σ_C .
: “BIGGER IS BETTER”.
: DESIGN FUNCTION.
: PREFERENCE ORDERING.

$$T \succ T^* \quad \text{IF} \quad \hat{h}(T, \sigma_C) > \hat{h}(T^*, \sigma_C)$$

¶ WHAT θ -MASS DESIGN MAXIMIZES THE ROBUSTNESS,
WITH DESIGN REQUIREMENT σ_C ?

THE LINEAR TAPER WHICH MIN-MAXes THE STRESS:

$$\hat{T}_\theta(X) = \frac{2\theta(L - X)}{L^2}$$

¶ GOOD NEWS or BAD NEWS?

3 INFO-GAP ROBUST-OPTIMAL DESIGN

¶ ROBUSTNESS OF THE OPTIMAL TAPER:

$$\hat{h}(\hat{T}_\theta, \sigma_C) = \frac{4W\theta^2 \sigma_C}{3L^4} - \tilde{\phi}$$

¶ TRADE-OFF:

ROBUSTNESS TO UNCERTAINTY, \hat{h} ,

VS.

PERFORMANCE, σ_C .

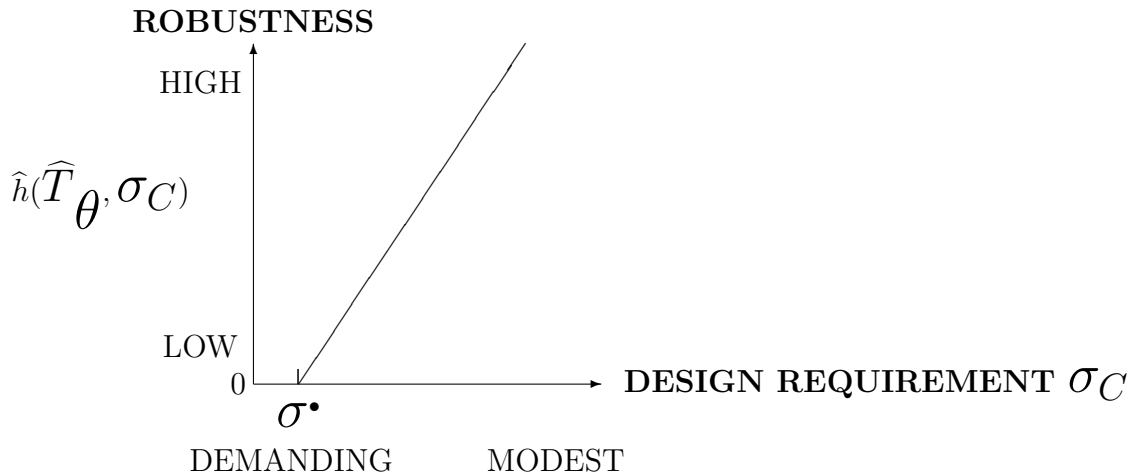


Figure 3: OPTIMAL ROBUSTNESS CURVE, $\hat{h}(\hat{T}_\theta, \sigma_C)$, VERSUS THE MAXIMUM-STRESS DESIGN REQUIREMENT σ_C .

¶ INFEASIBLE PERFORMANCE-ASPIRATION:

$$\sigma^\bullet = \frac{3\tilde{\phi}L^4}{4W\theta^2}$$

¶ INFEASIBLE PERFORMANCE-ASPIRATION:

$$\sigma^* = \frac{3\tilde{\phi}L^4}{4W\theta^2}$$

¶ PARETO-EFFICIENT MIN-MAX BENDING STRESS:

$$R(\hat{T}_\theta) = \frac{3\tilde{\phi}L^4}{4W\theta^2}$$

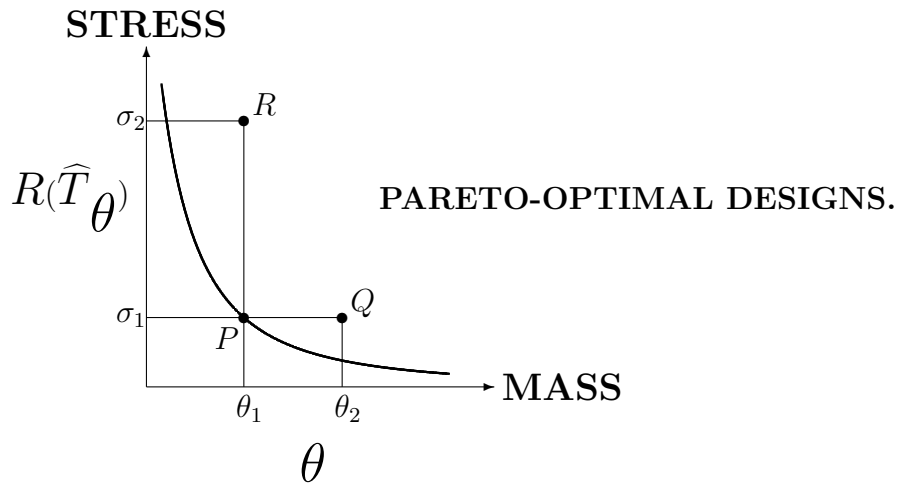


Figure 4: PARETO-OPTIMAL DESIGNS.

¶ CLASH of PERFORMANCE and ROBUSTNESS:

- ANY DESIGN ON THE PARETO SURFACE, E.G. P , HAS NO IMMUNITY TO LOAD UNCERTAINTY.
- ONLY SUB-OPTIMAL DESIGNS, E.G. Q , R , ARE FEASIBLE.
- 'GOOD' IS PREFERABLE TO 'BEST'.

4 ROBUSTNESS, SUB-OPTIMAL DESIGN and SAFETY FACTOR

¶ SUB-OPTIMAL DESIGN:

$$T^s(X) = \widehat{T}_\theta(X) + \gamma, \quad \gamma > 0$$

¶ WITH MAX-STRESS ASPIRATION:

$$\sigma^\bullet = \frac{3\tilde{\phi}L^4}{4W\theta^2}$$

¶ ROBUSTNESS OF SUB-OPTIMAL DESIGN IS:

$$\hat{h}(T^s, \sigma^\bullet) = \frac{\tilde{\phi}L^2}{4\theta^2} \left(\frac{2\theta}{L} + \gamma \right)^2 - \tilde{\phi}$$

¶ γ CONTROLS:

- SUB-OPTIMALITY of DESIGN.
- POSITIVITY of ROBUSTNESS.
- Related to classical “SAFETY FACTOR”.

¶ RECALL:

- h = HORIZON OF LOAD UNCERTAINTY.

- INFO-GAP MODEL:

$$\mathcal{U}(h, \tilde{\phi}) = \left\{ \phi(X) : \left| \phi(X) - \tilde{\phi}(X) \right| \leq h \right\}, \quad h \geq 0$$

- $\hat{h}(T^S, \sigma^\bullet) = \text{MAX } h \text{ SUCH THAT } \sigma \leq \sigma^\bullet \text{ FOR ALL LOADS.}$

¶ SAFETY FACTOR:

$F = \text{FRACTION of NOMINAL LOAD, } \tilde{\phi}.$

DEMANDED ROBUSTNESS:

$$\hat{h}_D = F \tilde{\phi}.$$

CHOICE OF γ SO THAT $\hat{h}(T^S, \sigma^\bullet) = \hat{h}_D$:

$$\gamma = \frac{2\theta}{L} \left(\sqrt{1+F} - 1 \right)$$

¶ OPTIMAL DESIGN:

$F = 0$: NO SAFETY FACTOR.

$\hat{h} = 0$ NO ROBUSTNESS.

$\gamma = 0$ PARETO-EFFICIENCY.

¶ SUB-OPTIMAL DESIGN:

$F > 0$ SAFETY.

$\hat{h} > 0$ ROBUSTNESS.

$\gamma > 0$ MATERIAL INEFFICIENCY.

5 OPPORTUNENESS FUNCTION

- TWO FACES OF UNCERTAINTY:
 - PERNICIOUS: POTENTIAL FOR FAILURE.
 - PROPITIOUS: POTENTIAL FOR WINDFALL.
- TWO MEASURES OF PERFORMANCE:
 - ROBUSTNESS: IMMUNITY FROM UNCERTAINTY.
 - OPPORTUNENESS: POTENTIAL FROM UNCERTAINTY.
- TWO STRATEGIES FOR DECISION:
 - SATISFICING: ENHANCE ROBUSTNESS. (\hat{h})
 - WINDFALLING: EXPLOIT OPPORTUNITY. ($\hat{\beta}$)

¶ INFO-GAP MODEL FOR UNCERTAIN LOAD:

$$\mathcal{U}(h, \tilde{\phi}) = \left\{ \phi(X) : \left| \phi(X) - \tilde{\phi}(X) \right| \leq h \right\}, \quad h \geq 0$$

¶ ROBUSTNESS OF DESIGN $T(X)$:

$$\hat{h}(T, \sigma_C) = \max \left\{ h : \max_{\phi \in \mathcal{U}(h, \tilde{\phi})} \max_{0 \leq X \leq L} |\sigma_{\phi, T}(X)| \leq \sigma_C \right\}$$

MAX h WITHOUT FAILURE FOR DESIGN SPEC. σ_C .

¶ DESIGN ASPIRATIONS:

σ_C = GREATEST ACCEPTABLE STRESS.

σ_W = VERY LOW AND DESIRABLE STRESS. “WINDFALL”.

$\sigma_W \ll \sigma_C$

¶ OPPORTUNENESS OF DESIGN $T(X)$:

MIN h AT WHICH WINDFALL IS POSSIBLE.

$$\hat{\beta}(T, \sigma_W) = \min \left\{ h : \min_{\phi \in \mathcal{U}(h, \tilde{\phi})} \max_{0 \leq X \leq L} |\sigma_{\phi, T}(X)| \leq \sigma_W \right\}$$

¶ OPPORTUNENESS FUNCTION, $\hat{\beta}(T, \sigma_W)$, IS AN IMMUNITY:

• IMMUNITY TO WINDFALL PERFORMANCE.

• “BIG IS BAD” FOR $\hat{\beta}(T, \sigma_W)$:

$$T \succ_o T^* \quad \text{IF} \quad \hat{\beta}(T, \sigma_W) < \hat{\beta}(T^*, \sigma_W)$$

¶ ROBUSTNESS FUNCTION, $\hat{h}(T, \sigma_C)$, IS AN IMMUNITY:

• IMMUNITY TO FAILURE.

• “BIGGER IS BETTER” FOR $\hat{h}(T, \sigma_C)$:

$$T \succ_R T^* \quad \text{IF} \quad \hat{h}(T, \sigma_C) > \hat{h}(T^*, \sigma_C)$$

¶ DOES THE ROBUSTNESS PREFERENCE, γ_R ,
 AGREE with the OPPORTUNENESS PREFERENCE $_{\phi}$?
 NOT NECESSARILY.

THE IMMUNITY FUNCTIONS MAY BE EITHER
 SYMPATHETIC OR ANTAGONISTIC.

¶ IN THE PRESENT BEAM EXAMPLE:

$$\widehat{\beta}(T, \sigma_W) = - \underbrace{\frac{\sigma_W}{\sigma_C} \widehat{h}(T, \sigma_C)}_A + \underbrace{\left(1 - \frac{\sigma_W}{\sigma_C}\right) \widetilde{\phi}}_B$$

¶ SYMPATHETIC IMMUNITIES:

- B DOES NOT DEPEND UPON DESIGN, $T(X)$.
- A IS POSITIVE.
- ANY CHANGE IN $T(X)$ WHICH
 IMPROVES the ROBUSTNESS (\widehat{h} GROWS),
 ALSO
 IMPROVES the OPPORTUNENESS ($\widehat{\beta}$ DIMINISHES).

¶ MANY EXAMPLES OF ANTAGONISTIC IMMUNITIES.
 E.G., WHEN “ B ” DOES DEPEND ON DECISION.

¶ IMMUNITY FUNCTIONS MAY CHANGE FROM
 ANTAGONISTIC TO SYMPATHETIC
 IN DIFFERENT DOMAINS OF THE PROBLEM.