

Dr. Yakov Ben-Haim
Professor
Yitzhak Moda'i Chair in
Technology and Economics



Technion
Israel Institute of Technology
Faculty of Mechanical Engineering
Haifa 32000 Israel

yakov@technion.ac.il
<http://www.technion.ac.il/yakov>
Tel: +972-4-829-3262 Fax: +972-4-829-5711
10 April 2007

A Response to Prof. Sniedovich

Prof. Moshe Sniedovich has written a short essay [7] entitled “The Fundamental Flaw in Info-Gap’s Uncertainty Model”. The mathematics of info-gap theory is very simple, and Prof. Sniedovich has the math right, but he is mistaken in the interpretation. I will explain his misunderstanding.

Prof. Sniedovich asks the following central question after presenting the formal definition of the info-gap robustness function (p.3):

In any case, the question is this: how should we interpret these measures of robustness? Are they of a global or local nature with regard to the region of uncertainty \mathcal{U} [the entire space of possibilities].

Indeed, interpretation is central here. The info-gap robustness function is “the degree of resistance to uncertainty and immunity to failure” ([1], p.38). When considering a specific decision or action, the robustness function is the answer to the question: by how much can our data and models err, and the contemplated action will still lead to an acceptable outcome? The robustness can be either small or large. If the robustness is large, then the decision is reliable. If the robustness is small, then the decision is unreliable. If the robustness is small for all available actions, then there is no responsible decision and one must re-think the problem, consider alternative concepts, acquire new data, etc.

Info-gap theory does not assert that, by evaluating the robustness to error in one’s information, one necessarily achieves large robustness. The robustness function evaluates the *degree* of resistance to uncertainty. Info-gap theory does not claim that “the solutions it generates are robust” as Prof. Sniedovich interprets (p.1).

Prof. Sniedovich also considers the issue of local versus global optimization. Prof. Sniedovich asserts (p.3) that info-gap “analysis is local *par excellence*” because the info-gap model of uncertainty, $\mathcal{U}(\alpha, \tilde{u})$, is centered on the estimated data and models \tilde{u} . The continuation of the earlier quote from Prof. Sniedovich’s essay (p.3) is:

Do they [the robustness] incorporate features of possible values of u in specific neighborhoods of \mathcal{U} [the entire space of possibilities]? Or are they based on representative values of u appropriately selected from \mathcal{U} to ensure that they adequately represent the whole of \mathcal{U} ?

This, of course, is a rhetorical question. The *Info-Gap* analysis is based on regions of \mathcal{U} in the immediate neighborhood of the estimate \tilde{u} .

I will make four points.

First, Prof. Sniedovich refers to \tilde{u} as a “single point estimate” (p.1), which is correct in some simple examples. However, an info-gap model of uncertainty can consider far more general and comprehensive error or uncertainty in underlying models and data. I will refer to \tilde{u} with this broader meaning. For instance, in evaluating the info-gap robustness of a neural network for fault diagnosis, one can consider the errors and idiosyncracies in the entire training set of data [6]. Another example is that one can evaluate the robustness to uncertainty in the shape of a probability density function, not only uncertainty in its parameters ([1], pp.88–91; [5]). Consequently, the robustness evaluates immunity to information which is far more comprehensive than a single point estimate.

Second, even when \tilde{u} is a single point estimate, the robustness evaluates the vulnerability of the decision to error in this estimate. If all we know is the single estimate, it is valuable to know whether or not we are vulnerable to error in that estimate.

Third, if the info-gap robustness is small, then only small errors in the underlying models and data, \tilde{u} , can be tolerated. That is, if the robustness is small then the contemplated decision is reliable only in “the immediate neighborhood of the estimate \tilde{u} ” as Prof. Sniedovich asserts. On the other hand, if the robustness is large, then the contemplated decision will result in acceptable outcome even if \tilde{u} errs greatly and the true values are far away from “the immediate neighborhood of the estimate \tilde{u} ”. Once again, the robustness can be either small or large; evaluating the robustness of a decision does not mean that the decision is robust. (What constitutes large robustness, and how robust is robust enough, are difficult questions dealt with in various parts of [1] and elsewhere, but this issue does not arise in Prof. Sniedovich’s essay.)

Fourth, info-gap theory is pluralistic and eclectic. It hybridizes easily with other theories of decision under uncertainty, both probabilistic and non-probabilistic. For instance, a Bayesian decision algorithm can be embedded in an info-gap analysis to evaluate the robustness to uncertainty in the prior probabilities. As another example, the outstanding work in robust optimization by my Technion colleagues Professors Ben-Tal and Nemirovski, mentioned approvingly by Prof. Sniedovich, can provide useful complementary insight into the immunity to uncertainty of a contemplated decision. Furthermore, the methods developed by Professors Ben-Tal and Nemirovski can be combined with an info-gap analysis in various ways.

I will summarize briefly why info-gap robustness is useful as a tool for decision-making under severe uncertainty. In many domains of activity, including engineering design, economic policy formulation, biological conservation, medical decisions, homeland security, project management, and so on, our information and understanding includes quantitative data and models. However, these data and models are often incomplete and erroneous, and even when they truthfully and comprehensively represent the past, the future may be fundamentally different. In such situations the decision maker can ask: for a given contemplated action, how wrong can my data and models be, and that action will still yield adequate results? The info-gap robustness function answers this question, and provides a quantitative tool (along with other tools where appropriate) for supporting a robust-satisficing decision strategy.

An additional, more fundamental motivation for info-gap robust-satisficing has begun to emerge, first suggested by an ethological application. It was found that the foraging behavior of a vast array of animal taxa is not readily described by energy-maximizing strategies, and seems to be described by info-gap robust-satisficing [4]. Why would such a strategy be preferred in competitive evolution? A possible answer, which is indicated by more recent theoretical analysis [2], seems to be that the info-gap robustness is a proxy for the probability of adequate outcome. That is, any change in action which augments the robustness, also augments the probability of achieving acceptable performance. In competitive environments the agents don’t need to optimize; they only need to beat the competition. For instance, some foraging animals only need enough food to last until the next foraging session. The proxy theorem indicates that robust-satisficing foraging strategies maximize the probability of gaining enough food; optimizing strategies can have lower probabilities for attaining sufficient food and thus will be removed during competition. If this speculative interpretation of the proxy theorem is correct, then we are a step closer to understanding why robust-satisficing provides concise explanations of economic conundrums such as the home bias paradox [3] and the equity premium puzzle [1, section 11.5], as well as behavioral observations such as the Ellsberg and Allais paradoxes [1, sections 11.1, 11.2].

I will conclude by very briefly mentioning the info-gap opportuneness function [1]. Uncertainty can be pernicious and threaten failure, which motivates the info-gap robustness function. However, uncertainty can also be propitious and offer the possibility of sweeping, windfall success far in excess of the decision maker’s anticipations. This favorable potential of uncertainty motivates the info-gap opportuneness function which assesses the potential for windfall inherent in decision under uncertainty.

References

1. Ben-Haim, Yakov, 2006, *Info-Gap Decision Theory: Decisions Under Severe Uncertainty*, 2nd edition, Academic Press, London.
2. Ben-Haim, Yakov, 2007, Robust-satisficing and the probability of survival, De Nederlandsche Bank Working Paper Series, No.138.
3. Ben-Haim, Yakov and Karsten Jeske, 2003, Home-bias in financial markets: Robust satisficing with info-gaps, Federal Reserve Bank of Atlanta, Working Paper Series, 2003-35, Dec. 2003. SSRN abstract and full paper at: <http://ssrn.com/abstract=487585>.
4. Carmel, Yohay and Yakov Ben-Haim, 2005, Info-gap robust-satisficing model of foraging behavior: Do foragers optimize or satisfice?, *American Naturalist*, 166: 633–641.
5. Fox, David R., Yakov Ben-Haim, Keith R. Hayes, Michael McCarthy, Brendan Wintle, Piers Dunstan, 2007, An info-gap approach to power and sample size calculations, *Environmetrics*, vol. 18, pp.189–203.
6. Pierce, S.Gareth, Yakov Ben-Haim, Keith Worden, Graeme Manson, 2006, Evaluation of neural network robust reliability using information-gap theory, *IEEE Transactions on Neural Networks*, vol.17, No.6, pp.1349–1361.
7. Sniedovich, Moshe, 2006, The Fundamental Flaw in Info-Gap's Uncertainty Model, Working Paper No. MS-3-06, Department of Mathematics and Statistics, The University of Melbourne.