What's wrong with this criticism

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I point out that in spite of recent claims to the contrary, the proof of Bell's theorem in Asher Peres's book works even in the presence of time-correlated hidden variables in the detectors.

One of the endearing traits of Asher Peres is that when somebody publishes something he knows to be wrong, he does not bother to refute it, even if the paper criticizes his own work. Life is too brief for such frivolity. As a small 70th birthday present I would like to answer one such recent attack. It's not much of a present, since Asher will not read my paper. Why should he? He already knows this criticism is nonsense. But somebody has to set the written record straight for future historians, so I will do it as part of this celebration. Fortunately this particular issue is so easily settled that this can be a very short paper. Since Asher is a master of the *very* short paper, my Peresian brevity is an important part of my act of homage. The criticism I address can be found in a new formulation by Karl Hess and Walter Philipp of their view that all versions of Bell's theorem are fundamentally flawed.[1] I focus here only on their criticism of the version in Asher's book.[2]

We have N pairs of spin-$\frac{1}{2}$ particles in the singlet state. We wish to test the postulate that the result $a_{\mathbf{x}}(j) = \pm 1$ (in appropriate units) of measuring the spin of one member of the j-th pair along any direction $\mathbf{x}$ is predetermined by hidden variables, and similarly for the result $b_{\mathbf{y}}(j)$ of a spin measurement of the other member of the pair. Asher (along, of course, with many others) notes that if these predetermined values exist for all directions, then the quantity

$$\gamma_j = a_{\mathbf{a}}(j)b_{\mathbf{b}}(j) + a_{\mathbf{a}}(j)b_{\mathbf{c}}(j) + a_{\mathbf{d}}(j)b_{\mathbf{b}}(j) - a_{\mathbf{d}}(j)b_{\mathbf{c}}(j), \tag{1}$$

constructed out of four such predetermined values, is constrained to have the value 2 or -2. If we average uniformly over all N pairs, we have

$$-2 \leq \gamma = (1/N)\sum_j \gamma_j = (1/N)\sum_j a_{\mathbf{a}}(j)b_{\mathbf{b}}(j) + (1/N)\sum_j a_{\mathbf{a}}(j)b_{\mathbf{c}}(j) +$$
$$(1/N)\sum_j a_{\mathbf{d}}(j)b_{\mathbf{b}}(j) - (1/N)\sum_j a_{\mathbf{d}}(j)b_{\mathbf{c}}(j) \leq 2. \tag{2}$$

But for large enough N, each of the four sums of products appearing in (2) is given to arbitrary precision by a quantum mechanical expectation value, and these expectation values, for suitable choice of measurement directions $\mathbf{a}, \mathbf{d}$ for one particle and $\mathbf{b}, \mathbf{c}$ for the other, can violate (2) by as much as 40%. This demonstrates, in Asher's immortal words,[3]

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[1] Karl Hess and Walter Philipp, PNAS, **101**, 1799-1805 (2004).

[2] Asher Peres, *Quantum Theory: Concepts and Methods*, Kluwer Academic Publishers, 1993, p. 164. I plan to comment more generally on Ref. 1 elsewhere.

[3] A. Peres, American Journal of Physics 46, 745-747.

that "unperformed experiments have no results."

Hess and Philipp point out a gap in the argument. I would put it this way: since each pair can be measured along only a single one of the four possible pairs of directions, the actual data one collects from a series of measurements on N pairs do not enable one to construct the quantity γ appearing in (2) but only the quantity

$$\gamma_{\rm exp} = (1/N_{\bf ab}) \sum_{j\in X_{\bf ab}} a_{\bf a}(j) b_{\bf b}(j) + (1/N_{\bf ac}) \sum_{j\in X_{\bf ac}} a_{\bf a}(j) b_{\bf c}(j) +$$

$$(1/N_{\bf db}) \sum_{j\in X_{\bf db}} a_{\bf d}(j) b_{\bf b}(j) - (1/N_{\bf dc}) \sum_{j\in X_{\bf dc}} a_{\bf d}(j) b_{\bf c}(j), \tag{3}$$

where $N_{\bf xy}$ is the number of pairs for which the detectors were set to $\bf x$ and $\bf y$ at the time of detection, and $X_{\bf xy}$ is the set of those pair indices j associated with such runs. Since the sets $X_{\bf ab}$, $X_{\bf ac}$, $X_{\bf db}$, and $X_{\bf dc}$ are mutually disjoint, (3) contains only a single product of predetermined values for every pair j. Thus none of the bounded quantities γ_j in (1), which all involve *four* theoretical predetermined values associated with the *same* run, appear in (3). So the argument that $\gamma_{\rm exp}$ must be bounded in magnitude by 2 "comes to a halt". Hess and Philipp conclude that if there is an argument that actual data are incompatible with predetermined values, it has yet to be made.

What they overlook, however, is a very simple way past this objection which is surely what Asher, who has a well known distaste for being explicit about what should be obvious,[4] had in mind. In each run the choices of whether $\bf x$ is $\bf a$ or $\bf d$ and whether $\bf y$ is $\bf b$ or $\bf c$ are made randomly and independently at each detector. As a result, for each of the four choices for $\bf xy$ the $N_{\bf xy}$ indices j appearing in $X_{\bf xy}$ constitute a random sample of the full set of indices $j = 1 \ldots N$, each j having a probability $\frac{1}{4}$ of appearing in $X_{\bf xy}$. So by standard sampling arguments, when N is large each term in $\gamma_{\rm exp}$ will be very close to the corresponding term in γ, with a variance that is negligible for large enough N. If predetermined values exist for each pair, then for large enough N the experimentally determined quantity $\gamma_{\rm exp}$ must, with overwhelming probability, also be bounded by 2, except for minute fluctuations, much less than the 40% deviation theory gives for the choice of measurement directions ("settings") in the Aspect experiments. QED.

Early in their paper Hess and Philipp adopt an assumption (*iii*) that "the average of the actual data measured for ... a given pair of settings equals the average that would be obtained for all particles if they had been measured with the same pair of settings." Why then do they overlook this elementary way to restart their "halted" proof? Their oversight seems to have arisen out of their continuing interest in a local hidden-variables model that they had earlier claimed[5] reproduced the quantum mechanical correlations. This model

[4] This can be disconcerting when, unlike in the present case, it is obvious only to him.

[5] Karl Hess and Walter Philipp, PNAS, **98**, 14224-14227, 14228-14233 (2002). This earlier claim is not repeated in Ref. 1.

is based on hidden variables λ that both particles acquire at the source, and additional hidden variables $\lambda^*(t)$ at one detector and $\lambda^{**}(t)$ at the other, which determine how the source hidden variable λ is processed at each detector to compute all the predetermined outcomes $a_{\mathbf{x}}(j)$ or $b_{\mathbf{y}}(j)$. Because λ^* and λ^{**} depend on time, the form of this processing can vary with the common time t at which both members of each pair are measured.[6] When Hess and Philipp discuss Asher's argument, their attention is focused on the roles played by λ, $\lambda^*(t)$, and $\lambda^{**}(t)$. They first consider the case in which there are no detector variables and the number of values of the source variable λ is finite or countably infinite. They show, by grouping together terms according to the value of the source variable, that the argument can still be justified. But their attempt to proceed along these lines "no longer works" when the processing of λ into predetermined outcomes is governed by time dependent detector variables $\lambda^*(t)$ and $\lambda^{**}(t)$.

But the simpler and more general sampling argument does work. It requires only the existence of predetermined values and is quite independent of the local mechanism that produces them. How did Hess and Philipp miss it? My guess is that their long-standing fascination with outcome correlations mediated by time-correlated detector variables, which they maintain physicists have overlooked for the past forty years,[7] blinded them to this elementary and more broadly based proof.

So, Future Historian, when you assess the contributions to science of Asher Peres in the 20th century and (I very much hope) well into the 21st, do not include among his sins against clear thinking, alleged errors arising from his neglect of the possible consequences of time-correlated detector hidden variables.

Acknowledgments. Supported by the National Science Foundation, Grant No. 0098429. I would like to thank Anton Zeilinger and the Schrödinger Institute for their hospitality in Vienna in October and November, 2003. I thank Karl Hess for sending me a pre-publication version of Ref. 1 and for criticizing my attempts, from Vienna and from Ithaca, to explain to him what was wrong with it. I thank Richard Gill, for providing reassuring reality checks from a card-carrying probabilist. And, needless to say, I am grateful to my friend Asher Peres for wonderful conversations and emails over many many years.

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[6] The fact that the two detections can take place at independently chosen times without altering the outcome probabilities is in itself fatal for the ability of such a model to reproduce the correlations given by quantum mechanics. I discuss this elsewhere.

[7] This claim is also puzzling. Hess and Philipp are aware that John Bell invokes such variables, in his well-known disquisition on the oddly correlated socks of R. A. Bertlemann (Journal de Physique, Colloque C2, suppl. au numero 3, **42**, C2 41-61 (1981)). They occur in his remarks about the influence of Sundays on the daily incidence of heart attacks in Lille and Lyons. But Hess and Philipp read these remarks as "discarding time-related parameters", rather than as citing them as an example of the kinds of classical sources of correlation that Bell has in mind.