

# Derivations of universal exact complex absorption potentials by the generalized complex coordinate method

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**Abstract.** On the basis of the Moiseyev–Hirschfelder generalization of the complex coordinate method, a universal energy-independent complex absorbing potential (CAP), is derived. It is proven that the universal CAP consists of flux and diffusion-type operators. When a smooth exterior scaling is used, the CAP gets non-zero values in the region where the interaction potential vanishes. An illustrative numerical example is given where narrow and broad, isolated and overlapping resonances were all calculated with more than nine digits of accuracy.

## 1. Introduction

In multichannel problems one can define a non-local, energy-dependent operator, so-called optical potential, that enables the calculation of cross sections for a subset of channels. All effects caused by the excluded channels are accounted for by the optical potential. ‘Optical potentials’ have been used in nuclear physics for a long time [1].

These types of optical potentials should be distinguished from negative-imaginary-short-range potentials which are added to the Hamiltonian in order to impose absorbing-boundary-conditions which provide outgoing waves in the asymptotic limit. In the literature these types of potentials are sometimes also called optical potentials [2]. To avoid confusion we do not use the term optical potential here but adopt the more suitable expression, complex absorbing potentials (CAPs), as proposed by Riss and Meyer [3] in order to name the artificial potentials introduced to impose absorbing boundary conditions. In molecular physics the use of CAPs avoids artificial reflections which result from the use of the finite basis/grid approximation [4] and allows simulations of large-scale strongly coupled scattering problems (such as in four-centre reactions) involving millions of basis functions [5]. In optical simulations the Maxwell equation is solved by using CAPs (see for example [6]) to design waveguides which have specific properties. More recently, Neuhauser [7] presented a new highly accurate and anomaly free time-independent approach to reactive scattering based on the use of very-short-range imaginary potentials. In a one-dimensional simulation the CAPs only covered two grid points!

Unlike other methods such as the complex-coordinate method which stays on a solid mathematical ground given by Balslev and Combes [8] and Simon [9], the use of CAPs was based on intuition and numerical experience. It has been proven that poles of the scattering matrix are also the eigenvalues of the complex-scaled Hamiltonian, but it has

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not been proven that they are the eigenvalues of the Hamiltonian which is perturbed by a CAP. Rom *et al* [10] have shown that the use of a CAP is similar to the use of the exterior scaling and more precise compared with use of the smooth-exterior-scaling method which was formulated by Rom *et al* [11] (the derivation of the smooth-exterior-scaling method is based on the Moiseyev–Hirschfelder representation of several complex coordinate methods by similarity transformation operators [12]). Riss and Meyer [13] also addressed themselves to the question of: Under what conditions are the resonances, obtained by using CAPs, actually the poles of the scattering matrix? Their strategy and derivation was as follows. The CAPs would provide the exact poles of the  $S$ -matrix if they could be reflection-free potentials. The reflection can be made smaller than any given limit, if the CAP is made weak and long enough. Moreover, they showed that one may minimize the reflections of a short-range CAP by adding specific energy-dependent terms to the effective Hamiltonian which includes the CAP. The thus modified CAP is in fact reflection free at one specific energy. Taking the CAP strength to be linearly dependent in the energy and transforming the above Hamiltonian by a similarity transformation they could considerably reduce the energy dependence. Their new effective Hamiltonian consists of a smooth-exterior-scaled kinetic operator and an energy-dependent potential term which has vanished (unlike the usually used CAPs) when the potential of interaction,  $V$ , has also vanished. Their so-called TCAP-method [13] is in fact very similar to the smooth exterior complex scaling (11) except that they add an extra local complex potential which is problem dependent.

The strategy taken in this paper is contrary to the Riss and Meyer one. Riss and Meyer started from the Hamiltonian perturbed by a CAP and ended up with a complex-scaled-type operator. We finish using the Moiseyev–Hirschfelder generalized representation of the complex coordinate method and ended up with the non-scaled Hamiltonian perturbed by a CAP which is problem independent. As we show in this paper this *universal energy-independent* CAP is a linear combination of flux and diffusion-type operators. In order to avoid confusion we should stress that the flux CAP derived here is not related to the use of reactive flux and CAPs made recently by Jäkel and Meyer [14] in the formulation of a new modified flux operator formalism for the calculations of state-to-state transition probabilities. It is also different from the Bloch flux-type CAP derived by Lipkin *et al* [15] from the exterior complex scaling approach [16]. The main difference between the Bloch-type CAP derived by Lipkin and coworkers and the CAP operator derived here, is in the fact that the Bloch operator is energy dependent whereas the CAP presented below (see equation (8)) is energy independent. Therefore, the use of the Bloch-type CAP enables the calculations of the resonances one by one and by the use of an iterative numerical procedure [15], whereas the use of the CAP derived here enables one to immediately obtain many resonances from a single diagonalization of a complex non-Hermitian matrix.

Moreover, from our derivation it is clear why *global* and local energy-dependent CAPs provide the resonances when there is a single dominant open channel to decay, and failed to do so in the cases where there are several dominant open channels for decaying. From our derivation one can clearly see why it is sufficient to use very short-range CAPs. The universal energy-independent CAP is applicable, however, to the general multichannel decay problem.

A simple illustrative numerical application of this new CAP to a one-dimensional model problem is given. For a given basis set all resonance poles which are explored by the conventional complex coordinate method are obtained by the new CAP operator in all significant figures. All resonance poles imply narrow and broad, isolated and overlapping poles of the scattering matrix.

## 2. Exact energy-independent universal CAP

The Moiseyev–Hirschfelder [12] generalization of the complex coordinate method associated the resonance poles of the  $S$ -matrix,  $E = E_r - iE_i$ , with the  $\theta$  independent complex eigenvalues of  $\hat{\mathcal{H}}$ :

$$\hat{\mathcal{H}}\Psi = E\Psi \tag{1}$$

where,

$$\hat{\mathcal{H}} = -\frac{\hbar^2}{2M} \frac{\partial^2}{\partial z^2} + V(z) \tag{2}$$

and  $z = F(x)$  is a path in the complex coordinate plane such that,

$$z = F(x) \rightarrow x \exp(i\theta), \quad \text{as } x \rightarrow \infty. \tag{3}$$

In the spirit of Simon’s proposition, to avoid the need of carrying out analytical continuation of the potential term in the Hamiltonian into the complex coordinate plane [16] Rom and coworkers proposed defining a smooth-exterior-scaling path which is defined as [12]:

$$f(x) = \frac{\partial F}{\partial x} = 1 + (\exp(i\theta) - 1)g(x) \tag{4}$$

where  $g(x)$  is varied from 0 to 1 around the point  $x = x_0$ . If  $V(x \geq x_0) = 0$  one can use the unscaled potential  $V(x)$  rather than using the complex potential  $V(z)$ . Note, however, that the path which defined in equation (4) is very general and is not necessarily limited to the case where  $V(z) = V(x)$  or when  $V(z) \sim V(x)$ .

It is easy to see that since,

$$\frac{\partial}{\partial z} = f^{-1}(x) \frac{\partial}{\partial x} \tag{5}$$

then,

$$\frac{\partial^2}{\partial z^2} = -f^{-3}(x) \frac{\partial f(x)}{\partial x} \frac{\partial}{\partial x} + f^{-2}(x) \frac{\partial^2}{\partial x^2}. \tag{6}$$

Consequently the smooth-exterior-scaled Hamiltonian is given by,

$$\hat{\mathcal{H}} = -\frac{\hbar^2}{2M} \frac{\partial^2}{\partial x^2} + V[F(x)] + \hat{V}_{\text{CAP}} \tag{7}$$

where,

$$\hat{V}_{\text{CAP}} = \frac{1}{2} V_1(x) \frac{\partial}{\partial x} + V_2(x) \frac{\partial^2}{\partial x^2} \tag{8}$$

and,

$$V_1(x) = \frac{\hbar^2}{M f^3(x)} \frac{\partial f(x)}{\partial x} \tag{9}$$

$$V_2(x) = \frac{\hbar^2}{2M} (1 - f^{-2}(x)). \tag{10}$$

The volume element is given by,

$$dz = f(x)dx. \tag{11}$$

As usual one can transform the Hamiltonian in order to simplify the expression of the volume element to be equal to  $dz = dx$  by defining a new function  $\Phi$ ,

$$\Psi(x) = f^{-\frac{1}{2}} \Phi(x) \tag{12}$$

such that,

$$\hat{\mathcal{H}}_f = -\frac{\hbar^2}{2M} \nabla_f^2 + V[F(x)] \tag{13}$$

where,

$$\nabla_f^2 = f^{+\frac{1}{2}}(x) \frac{\partial^2}{\partial z^2} f^{-\frac{1}{2}}(x).$$

After some algebraic derivations one finds that  $\Phi(x)$  is an eigenfunction of  $\hat{\mathcal{H}}_f$ ,

$$\hat{\mathcal{H}}_f = -\frac{\hbar^2}{2M} \frac{\partial^2}{\partial x^2} + V[F(x)] + \hat{V}_{\text{CAP}}^{(f)} \tag{14}$$

where,

$$\hat{V}_{\text{CAP}}^{(f)} = V_0(x) + V_1(x) \frac{\partial}{\partial x} + V_2(x) \frac{\partial^2}{\partial x^2}. \tag{15}$$

The functions  $V_1(x)$  and  $V_2(x)$  are as defined in equations (9) and (10) where  $V_0(x)$  is given by,

$$V_0(x) = \frac{\hbar^2}{4M} f^{-3}(x) \frac{\partial^2 f}{\partial x^2} - \frac{5\hbar^2}{8M} f^{-4}(x) \left( \frac{\partial f}{\partial x} \right)^2. \tag{16}$$

As one can see, the consequence of simplifying the expression of the volume element is in the inclusion of an extra term  $V_0$  in the CAP and in doubling the weight of the flux-type term.

Without loss of generality let us define a specific family of integration paths in the complex coordinate plane by defining  $g(x)$  in equation (4) as:

$$g(x) = 1 + 0.5(\tanh(\lambda(x - x_0)) - \tanh(\lambda(x + x_0))). \tag{17}$$

By integrating over  $g(x)$ , the complex paths,  $F(x)$ , are obtained,

$$F(x) = x + (e^{i\theta} - 1) \left[ x + \frac{1}{2\lambda} \ln \left( \frac{\cosh[\lambda(x - x_0)]}{\cosh[\lambda(x + x_0)]} \right) \right]. \tag{18}$$

Illustrative examples for different possible integration paths are given in figure 1 for  $\lambda = 5$  and when  $x_0 = 0$  or when  $x_0 = 6$ . For  $x_0 = 0$  (or when  $\lambda = 0$ ) the usual complex coordinate path,  $z = x \exp(i\theta)$ , is obtained. For large values of  $\lambda$  the smooth-exterior-scaling path as proposed by Rom and coworkers [12] is obtained. At the limit of  $\lambda \rightarrow \infty$  the exterior scaling path [16] is obtained and,

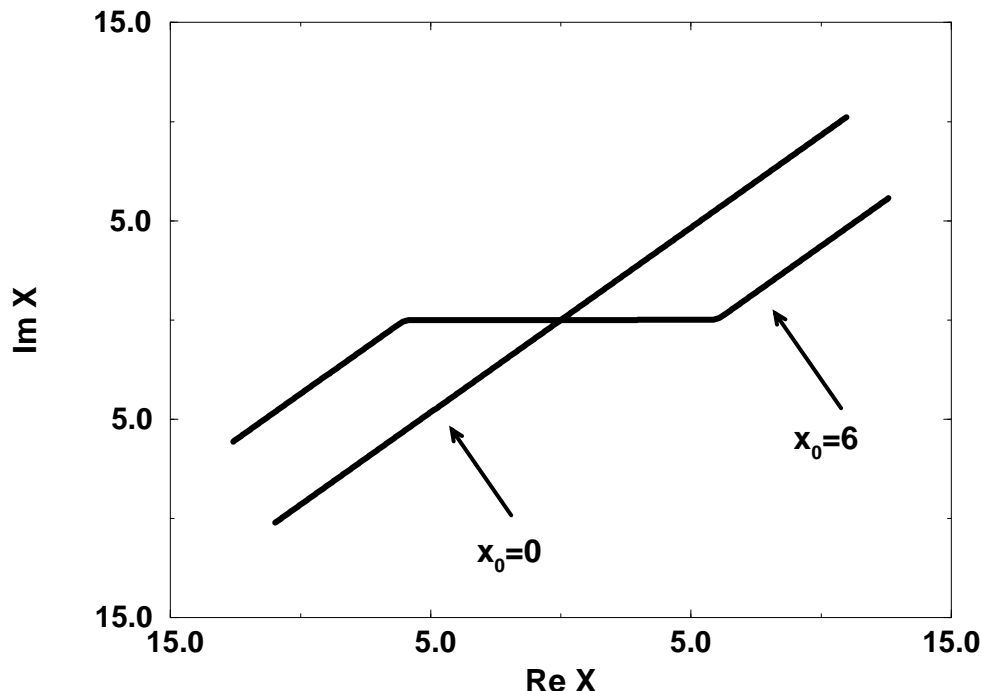
$$z = \begin{cases} x & \text{if } -x_0 \leq x \leq x_0 \\ (x - x_0)e^{i\theta} + x_0 & \text{if } x > x_0 \\ (x + x_0)e^{i\theta} - x_0 & \text{if } x < -x_0. \end{cases} \tag{19}$$

The CAP terms can be calculated using the above expression for  $g(x)$  and the following analytical expressions for the first and second derivatives of  $f(x)$ :

$$\frac{\partial f}{\partial x} = 0.5\lambda(\exp(i\theta) - 1)(\tanh^2(\lambda(x + x_0)) - \tanh^2(\lambda(x - x_0))) \tag{20}$$

and,

$$\begin{aligned} \frac{\partial^2 f}{\partial x^2} = & \lambda^2(\exp(i\theta) - 1)(\tanh(\lambda(x + x_0))(1 - \tanh^2(\lambda(x + x_0))) \\ & - \tanh(\lambda(x - x_0))(1 - \tanh^2(\lambda(x - x_0)))). \end{aligned} \tag{21}$$



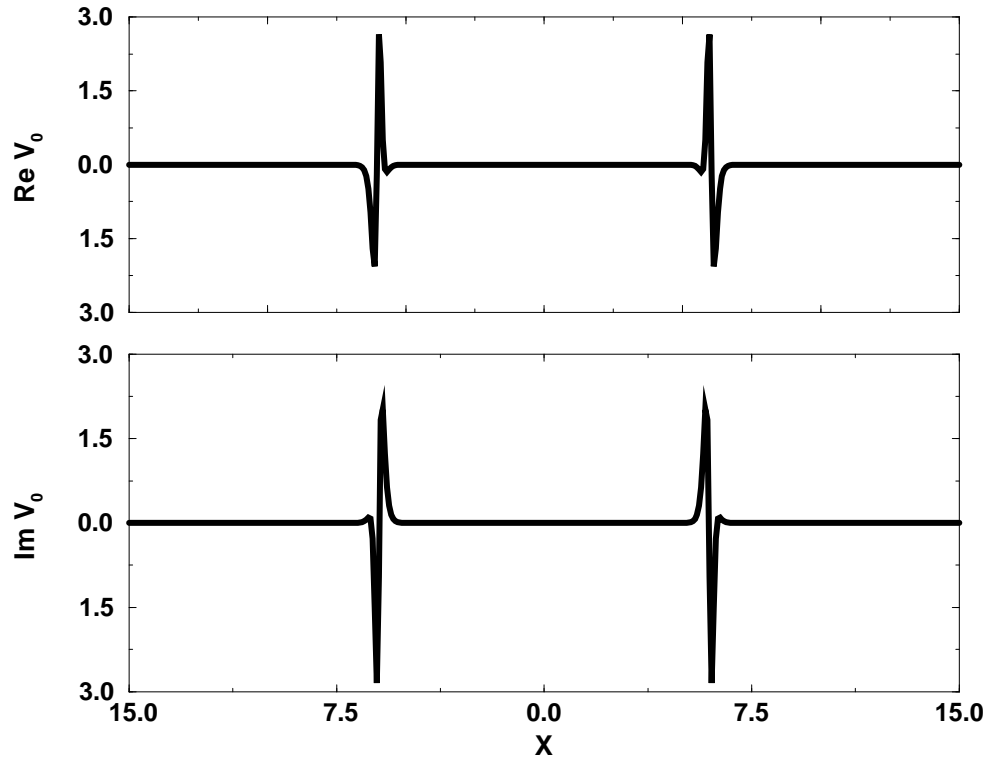
**Figure 1.** Several possible integration paths,  $F(x)$ , in the complex coordinate plane (see equation (18)). For  $x_0 = 0$  the conventional complex path is obtained, for  $x_0 = 6$  a smooth-exterior-scaling path is obtained. In both cases  $\theta = 0.75$  rad and  $\lambda = 5$  au.

In figures 2–4 illustrative examples for  $V_0$ ,  $V_1$  and  $V_2(x)$  are given for  $\lambda = 5$ . As one can see from figure 3,  $V_1(x)$  is an extremely short-ranged potential for the chosen value of  $\lambda$ . Since  $V_1(x)$  looks like a delta function and since the flux operator is defined as  $i\delta(x - x_0)\frac{\partial}{\partial x}$  we refer to the corresponding first term in the CAP which is defined in equation (8) as a flux-type operator. The second term is a kinetic-type operator which describes the diffusion at  $x \sim x_0$ . As one can see from figure 4,  $V_2(x)$  vanishes when  $-x_0 < x < x_0$ . The value of  $x_0$  can be chosen such that only within this interval of  $x$  can the physical potential get non-zero values and vanish elsewhere.

As we will show later, the use of the universal flux-diffusion-type CAP which is constructed from the  $V_1$  and  $V_2$  functions, presented in figures 3 and 4, enables us to obtain, at a very high accuracy, many resonances (regardless of their widths and being isolated or overlapping resonances) from a single diagonalization of a complex non-symmetric matrix. The width is defined as usual as  $\Gamma = -2E_i$ . It should be stressed that in our calculations the potential  $V(x)$  remained real and unscaled due to the properties of the chosen integration path,  $F(x)$ , (see figure 1 for  $\lambda = 5$ ).

### 3. Exact energy-dependent local and short-range CAP

As pointed out by Moiseyev and Hirschfelder [12], the integration paths in the complex coordinate plane should be taken such that  $z \rightarrow x \exp(i\theta)$  as  $x \rightarrow \infty$ . It implies that if  $V = 0$  (i.e. the threshold energy,  $E_t$ , is taken as a zero reference energy) when  $x \geq x_0$  or



**Figure 2.** The complex  $V_0(x)$  local potential (see equation (16)) which should be added to the universal energy-independent CAP when the Hamiltonian is transformed to make the volume element equal to  $dz = dx$ . The path in the complex coordinate plane is  $F(x)$  as shown in figure 1 for  $\lambda = 5$ .

when  $x \leq -x_0$ , then,

$$-\frac{\hbar^2 e^{-2i\theta}}{2M} \frac{\partial^2}{\partial x^2} \Psi = E \Psi \quad (22)$$

and therefore when  $x \leq -x_0$  or when  $x \geq x_0$ ,

$$\Psi(x) = a e^{+ik e^{i\theta} x} \quad (23)$$

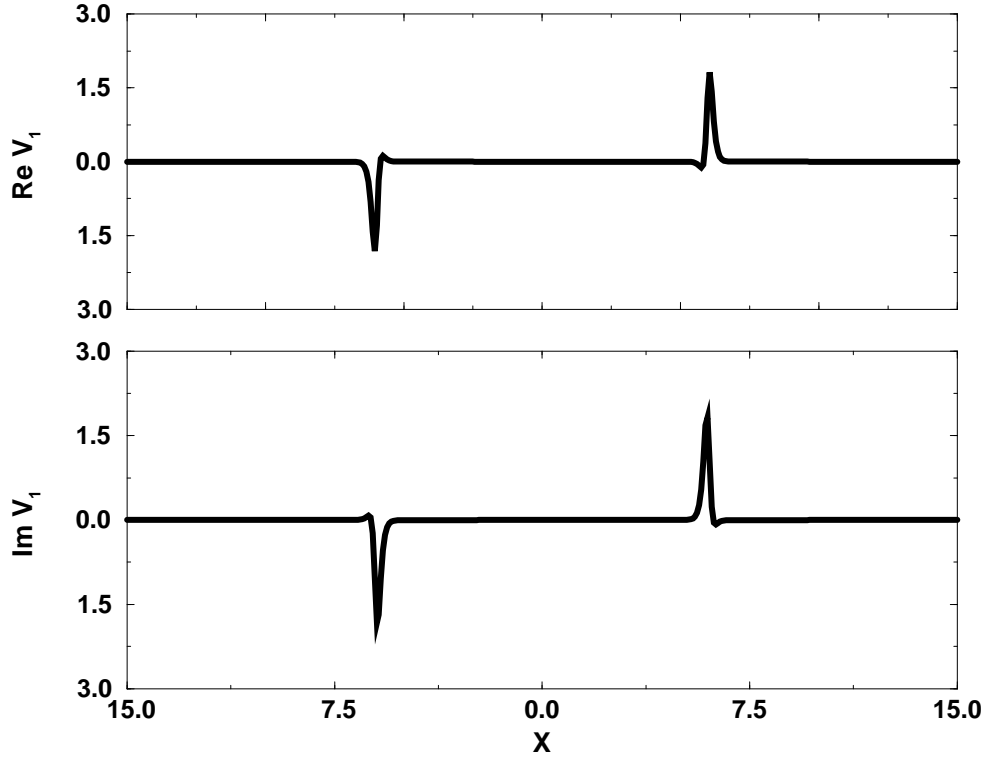
where,

$$k = \frac{[2ME]^{1/2}}{\hbar}. \quad (24)$$

By inserting equations (22)–(24) into equation (8) a local, but energy-dependent, CAP is obtained,

$$V_{\text{CAP}}^{[ED]}(x) = i \frac{\exp(i\theta) \sqrt{2ME}}{2\hbar} V_1(x) - \frac{2M \exp(i\theta)}{\hbar^2} V_2(x). \quad (25)$$

Since  $E$  in expression of (25) is associated with the eigenvalue  $E$  of  $\hat{H}(x) + V_{\text{CAP}}^{[ED]}(x)$ , only a single resonance can be obtained at the time and it is impossible to obtain the positions and widths of several resonances from a single diagonalization of the complex Hamiltonian. Moreover, the resonance solution can be obtained by carrying iterative calculations which



**Figure 3.** The complex  $V_1(x)$  local extremely short-range potential (see equation (9)) which is the coordinate-dependent linear factor of the flux-type term in  $\hat{V}_{\text{CAP}}$  (see equation (8)). The path in the complex coordinate plane is  $F(x)$  as shown in figure 1 for  $\lambda = 5$ .

may show a slow convergence, or by treating  $E$  in equation (25) as an independent parameter (denoted by  $E_{\text{CAP}}$ ). That is, choose some value for  $E^{\text{CAP}}$ ; diagonalize the Hamiltonian; insert  $E^{\text{CAP}} = E(\text{eigenvalue})$ ; and iterate to self-consistency,

$$|E - E_{\text{CAP}}| \leq \epsilon, \quad (26)$$

where  $\epsilon$  is a small number which depends on the machine accuracy ( $10^{-16}$  in double precision numerical calculations).

We can define a new complex parameter,  $\alpha = |\alpha| \exp(i\phi)$ , which replaces the parameter  $E_{\text{CAP}}$ ,

$$\alpha = e^{i\phi} \frac{\sqrt{2ME_{\text{CAP}}}}{\hbar}. \quad (27)$$

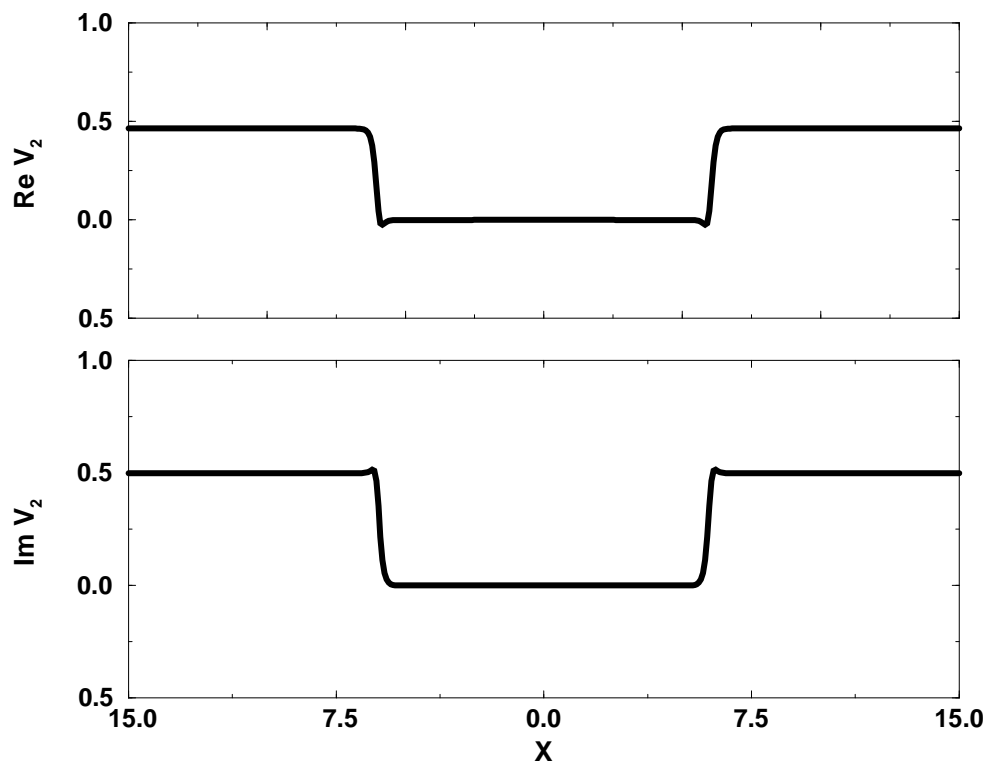
Therefore equation (25) can be rewritten,

$$V_{\text{CAP}}^{[ED]}(x) = i\frac{\alpha}{2}V_1(x) - \alpha^2V_2(x) \quad (28)$$

where  $\alpha$  is the complex parameter for which the following two equations should be satisfied:

$$\frac{\partial E}{\partial |\alpha|} = 0 \quad \frac{\partial E}{\partial \phi} = 0. \quad (29)$$

Note that although  $V_1(x)$  and  $V_2(x)$  depend on another parameter  $\theta$ , the resonance complex eigenvalue depends only on the value of  $\alpha$  (as in the usual complex coordinate method also



**Figure 4.** The complex  $V_2(x)$  potential which vanishes when the physical potential gets non-zero values (see equation (10)) which is the coordinate-dependent linear factor of the diffusion-like term in  $\hat{V}_{\text{CAP}}$  (see equation (8)). The path in the complex coordinate plane is  $F(x)$  as shown in figure 1 for  $\lambda = 5$ .

in the Moiseyev–Hirschfelder generalized version of the method the resonances eigenvalues are  $\theta$  independent).

Since the derivation of  $V_{\text{CAP}}^{[ED]}(x)$  is based on the validity of equation (23) it is clear that its use is limited to one-dimensional problems or multidimensional problems where there is a single dominant decaying channel. If this is not the case we should take into consideration the fact that for each one of the decaying channels we need to define another  $V_{\text{CAP}}^{[ED]}(x)$  where the parameter  $E$  in equations (25) and (27) is replaced by  $E - E_t^{(n)}$ .  $E_t^{(n)}$  is the threshold energy of the  $n$ th decaying channel. That is, for  $n$  open channels for decaying one needs to optimize  $n$  different  $\alpha_i$ ;  $i = 1, 2, \dots, n$  parameters. It is clear that in such a case the use of local energy-dependent CAPs is not simple and therefore we prefer to use the CAP as given in equation (8).

#### 4. Illustrative numerical example

The one-dimensional model Hamiltonian with a symmetrical double barrier potential,

$$\hat{H} = \hat{p}_x^2/2 + (x^2/2 - 0.8) \exp(-0.1x^2) + 0.8, \quad (30)$$

has been used before as a test case for new theories and computational algorithms for calculating resonances [17]. The poles of the scattering matrix in this case consist of one

bound state, two isolated resonances, and many broad overlapping resonances.

In our numerical calculations we used 500 Fourier functions,

$$|n\rangle = L^{-1} \exp(i2\pi nx/L); \quad n = -249, -248, \dots, -1, 0, 1, \dots, +249, 250 \quad (31)$$

where  $L = 30$ , as a basis set to represent the unscaled Hamiltonian  $\hat{H}$  and the universal energy-independent  $\hat{V}_{\text{CAP}}$  which is given in equation (8). The integration path has been defined as in equation (18) where  $\theta = 0.75$  (in order to also expose the very broad resonances),  $x_0 = 6$  (in order to leave the double-barrier potential unscaled since  $V(x) = 0$  when  $x \leq 0$  or  $x \geq 0$ ), and  $\lambda = 5$  (in order to get into the complex coordinate plane as close as possible to  $x = 6$  in order to keep  $V$  real). The  $\hat{V}_{\text{CAP}}$  terms were calculated by approximating the identity operator by the 500 Fourier functions. That is,

$$\langle n' | V_m \frac{\partial^m}{\partial x^m} | n \rangle = \langle n' | V_m | n \rangle \left[ i \frac{2\pi n}{L} \right]^m; \quad m = 1, 2. \quad (32)$$

All bound and resonance eigenvalues of the  $500 \times 500$  complex non-Hermitian matrix,  $\mathcal{H} = H + V_{\text{CAP}}$ , were obtained with a remarkable agreement of nine significant figures with the results which were obtained from the diagonalization of the complex scaled Hamiltonian matrix (which has been constructed from the same Fourier basis set). The results obtained by the use of the new CAP are given in table 1 for the first eight lowest lying states in the complex energy plane (i.e. labelled by the increasing order of the width,  $\Gamma = -2\text{Im}(E)$ ).

### 5. Concluding remarks

The question we addressed in this paper was as follows. What is the universal (i.e. problem-independent) expression of  $\hat{V}_{\text{CAP}}$  for which,

$$\hat{\mathcal{H}} = \hat{H} + \hat{V}_{\text{CAP}}$$

where  $\hat{\mathcal{H}} = \hat{H}(F(x))$ ,  $\hat{H}$  is the unscaled physical Hamiltonian and  $F(x)$  is a path in the complex coordinate plane?

The motivation is clear. From the generalized complex coordinate method it is known that the resonance poles of the scattering matrix are associated with complex eigenvalues of  $\hat{\mathcal{H}}$ . We can choose a path in the complex coordinate plane that leaves the physical potential unscaled, i.e.  $V(F(x)) = V(x)$  for finite-range potentials and  $V(F(x)) \sim V(x)$

**Table 1.** The bound state and resonance positions,  $E_r$ , and widths,  $\Gamma = -2\text{Im}(E_i)$ , obtained by using the universal energy-independent flux-diffusion-type CAP,  $V_{\text{CAP}} = -i0.5V_1(x)\hat{p}_x - V_2(x)\hat{p}_x^2$  (see equation (8) and figures 3 and 4 for  $V_1(x)$  and  $V_2(x)$ ) for a double barrier potential used before as a test model for new theories and computational methods [17].  $\Delta E_r$  and  $\Delta E_i$  are respectively the deviations of  $E_r$  and  $E_i$  from the real and the imaginary parts of the complex energies of the conventional complex-scaled Hamiltonian (i.e.  $x_0 = 0$ ) matrix.

$\nu$	$E_r$	$E_i$	$\Delta E_r$	$\Delta E_i$
0	0.502 040 3621	0.000 000 0000	0.000 000 0000	0.000 000 0000
1	1.420 970 9510	-0.000 058 2665	0.000 000 0000	0.000 000 0001
2	2.127 197 0730	-0.015 447 3189	0.000 000 0000	0.000 000 0001
3	2.584 582 8690	-0.173 750 7099	0.000 000 0000	0.000 000 0003
4	2.924 421 8880	-0.564 794 9852	0.000 000 0040	0.000 000 0028
5	3.255 486 1380	-1.111 531 5160	0.000 000 1290	0.000 000 0790
6	3.557 218 0640	-1.755 501 0750	0.000 002 2390	0.000 004 9810
7	3.824 514 4790	-2.487 554 4370	0.000 184 9750	0.000 108 2550

for long-range potentials. In such a case one can construct the unscaled Hamiltonian matrix elements by conventional computational algorithms and by using available program packages (for example the Gaussian code by which the neutral and ionized polyatomic molecular Hamiltonians can be constructed) and add to it the matrix which represents the universal  $\hat{V}_{\text{CAP}}$  (using the same basis set which has been used to represent  $\hat{H}$ ).

In this paper we have proved that  $\hat{V}_{\text{CAP}}$  is a linear combination of two terms. One term is a flux-type operator,  $0.5V_1(x)\frac{\partial}{\partial x}$ , where  $V_1(x)$  can be an extremely short-range delta-type function, and the second one is a diffusion-like operator,  $V_2(x)\frac{\partial^2}{\partial x^2}$  where  $V_2(x) = 0$  when the physical potential has non-zero values. Note that when  $\hat{V}_{\text{CAP}}$  is a linear combination of only two terms, the volume element is not  $dx$  but  $f(x) dx$ . In order to keep  $dx$  as the volume element one should add another CAP term,  $V_0(x)$ , to the Hamiltonian and double the weight of the first term. When one wishes to obtain the energy spectrum by diagonalizing a complex and symmetric Hamiltonian matrix there is a need to add the  $V_0$  term. Namely, for orthonormal basis functions,  $\phi_i(x)$ ;  $\int_{\text{All-space}} \phi_i^*(x)\phi_j(x) dx = \delta_{i,j}$ , the Hamiltonian matrix elements of  $\hat{\mathcal{H}} = \hat{H} + \hat{V}_{\text{CAP}}$  are given by,  $\int_{\text{All-space}} \phi_i^*(x)\hat{\mathcal{H}}\phi_j(x) dx$ .

By replacing the first and second derivatives (i.e.  $\hat{p}_x$  and  $\hat{p}_x^2$  operators) by a complex parameter a local CAP which is energy dependent is obtained.

As an illustrative numerical example we calculated the narrow and broad, isolated and overlapping resonances of a one-dimensional model Hamiltonian using the universal energy-independent CAP. The resonance positions and widths were found to be numerically exact.

The application of the universal CAP to many-body and multidimensional problems is under process.

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