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Multiphoton dissociation or ionization: Annihilation of discrete quasienergy states in strong electromagnetic fields

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Two different mechanisms of annihilation of discrete quasienergy states in strong electromagnetic fields are described. In the first mechanism, the annihilation of a metastable Floquet state results from laser-induced avoided crossing between two quasienergy levels. The second mechanism is basically a threshold effect where the ionization or dissociation energy of a specific bound state of the field-free Hamiltonian (bound state of the diagonal term in the Floquet Hamiltonian matrix in the general case) is equal to $n\hbar\omega$, where ω is the frequency of the time-dependent field and n is the number of absorbed photons. An illustrating numerical example is given.

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I. INTRODUCTION

The advent of intense lasers led to intensively experimental and theoretical studying of nonlinear effects in multiphoton-ionization processes [1,2]. A striking experimental example of a nonlinear dynamical effect is multiphoton ionization of xenon where the above-threshold ionization (ATI) energy spectra show series of isolated peaks separated by one-photon energy [3]. Suppression of the slow electron peak was observed as the field intensity a was increased [4]. It is a point of interest that although free electrons do not absorb photons, the multiphoton ionization branching ratio σ_{n+1}/σ_n can be estimated from a continuum-continuum transition matrix element [5],

$$\frac{\sigma_{n+1}}{\sigma_n}=a^2\frac{\pi^2}{2}|\langle\psi^{(0)}_{E+\hbar\omega n}|\hat{H}_1|\psi^{(0)}_{E+\hbar\omega(n+1)}\rangle|^2 , \qquad (1)$$

where $\{\psi^{(0)}\}$ are the continuum eigenfunctions of the field-free Hamiltonian $\hat{H}_0$, and the time-periodic Hamiltonian is given by $\hat{H}=\hat{H}_0+\hat{H}_1\cos(\omega t)$. The above equation shows that the multiphoton ionization processes will be more efficient than single-photon processes, and suppression of slow electron peaks in the ATI spectra will be observed as the field intensity is increased to provide $\sigma_{n+1}/\sigma_n>1$ [5,6].

A striking numerical example of the nonlinear dynamics of laser-induced processes is the creation of additional discrete states as the field strength is increased. Two different cases were studied. In the first case, additional adiabatic states are created resulting from laser-induced avoided crossings between resonant electronic-field surfaces in molecular systems [7]. In the second case the dressed potential may support more bound states than the original field-free potential [8–10].

Recently it has been shown that in the case of hydrogen atoms in a superintense, high-frequency laser field, the dressed potential reduces drastically the binding energy of the ground state [11] rather than increasing it.

In this work we study the annihilation of discrete quasienergy states when the dressed potential supports the *same* number of bound states as the original bare potential does. Two different mechanisms for annihilation of discrete quasienergy states are suggested in Sec. II. In the first mechanism a bound dressed state is pushed into the continuum of the free-field Hamiltonian resulting from a laser-induced avoided crossing between two discrete quasienergy states. Such avoided crossing is most likely to be obtained when two dressed states are degenerated,

$$E_i^{(0)}-E_j^{(0)}=n\hbar\omega , \qquad (2)$$

where $E_i^{(0)}$ and $E_j^{(0)}$ are two bound states of the bare Hamiltonian.

The second mechanism, discussed in Sec. II, is basically a threshold effect where one of the bound states of the field-free Hamiltonian is degenerated with the shifted laser-induced continuua. That is,

$$|E_i^{(0)}-E_{\rm thr}|=n\hbar\omega , \qquad (3)$$

where $E_{\rm thr}$ denotes the threshold energy. In Sec. III, a numerical example is given to illustrate the annihilation of discrete quasienergy states due to the two mechanisms described above. Concluding remarks are given in Sec. IV.

II. ANNIHILATION OF DISCRETE QUASIENERGY STATES IN STRONG LASER FIELDS

The bound states of a dissociative system become resonances when the electromagnetic field is turned on. The metastable (resonance) quasienergy states are the solutions of the time-dependent Schrödinger equation,

$$\hat{H}\psi_\alpha = i\hbar\frac{\partial\psi_\alpha}{\partial t}, \tag{4}$$

$$H(\mathbf{x},t) = \hat{H}(\mathbf{x},t+T), \quad T = 2\pi/\omega, \tag{5}$$

which satisfy the Floquet-Bloch theorem, such that

$$\psi_\alpha = e^{-i\varepsilon_\alpha t/\hbar}\Phi_\alpha(\mathbf{x},t) \tag{6}$$

and Φ_α is periodic in time,

$$\Phi_\alpha(\mathbf{x},t) = \Phi_\alpha(\mathbf{x},t+T). \tag{7}$$

The complex quasienergies ε_α are associated with the positions E_α and widths Γ_α of the resonance laser-induced states

$$\varepsilon_\alpha = E_\alpha - \frac{i}{2}\Gamma_\alpha \tag{8}$$

and

$$\tau_\alpha = \hbar/\Gamma_\alpha, \tag{9}$$

where τ_α denotes the lifetime. Upon complex scaling

$$\mathbf{x} \to \mathbf{x}e^{i\theta}, \tag{10}$$

the resonance quasienergy states become square integrable [12]. The quasienergy states ψ_α can be introduced by

$$\Psi_\alpha = \hat{U}(\mathbf{x},t)\Psi_\alpha(\mathbf{x},0) = \hat{U}\Phi_\alpha(\mathbf{x},0), \tag{11}$$

where $\hat{U}(\mathbf{x},t)$ is the time-evolution operator. By substituting Eqs. (6) and (7) in Eq. (11) one can immediately see that the quasienergy states at $t=nT$ are eigenfunctions of the complex-scaled time-evolution operator after one optical cycle,

$$\hat{U}(\mathbf{x}e^{i\theta},T)\phi_\alpha(\mathbf{x}e^{i\theta},0) = \lambda_\alpha\Phi_\alpha(\mathbf{x}e^{i\theta},0) \tag{12}$$

and the complex quasienergies mapped to a single Brillouin zone (i.e., a single optical cycle) are obtained [13],

$$\lambda_\alpha = e^{-i\varepsilon_\alpha T/\hbar}. \tag{13}$$

Note that the mapping to a single Brillouin zone results from the fact that the quasienergies are defined only modulo $2\pi/T$. The complex eigenvalues λ_α can be divided into two different sets. The first one is of the resonances that are θ independent. The distance of the point in the complex λ plane that is associated with a resonance state from the origin $\lambda=(0,0)$ is

$$|\lambda_\alpha| = e^{-\Gamma_\alpha T/\hbar} \tag{14}$$

and its deviation from unity is a visual measurement for the lifetime of a resonance state. For very narrow resonances $\Gamma_\alpha \approx 0$ and $|\lambda| \approx 1$, whereas for broad resonances $\Gamma_\alpha \gg 0$ and $0 < |\lambda_\alpha| \ll 1$.

The second class of complex eigenvalues, λ_α, are θ-dependent and are associated with the rotating continua. As we have proved on the basis of the Balselev-Combes theorem, the rotating continua solutions form a spiral in the complex λ plane [13]. The edge of the spiral is at $\lambda_\alpha = 1+i0$ [i.e., (1,0) in the complex λ plane] if the threshold energy of the field-free Hamiltonian is zero [13]. The core of the spiral is at $\lambda_\alpha = 0+i0$ [i.e., (0,0) in the complex λ plane] [13]. Since the edge and the core are the *only* points on the spiral which are θ independent, we expect that the annihilation of resonance quasienergies that are also θ independent will be obtained when

$$\lambda_\alpha(\text{resonance}) \to 0+i0 \quad [\text{the core } (0,0)] \tag{15}$$

or

$$\lambda_\alpha(\text{resonance}) \to 1+i0 \quad [\text{the threshold } (1,0)] \tag{16}$$

as the field intensity is increased. The question is if we can estimate the laser frequency and intensity at which a specific resonance quasienergy state will be eliminated. In order to answer this question, we shall expand the periodic complex scaled $\Phi_\alpha(\mathbf{x}e^{\theta},t)$ function in a Fourier series,

$$\Phi_\alpha = \sum_{n=-\infty}^{\infty} \phi_{n,\alpha}(\mathbf{x}e^{i\theta})e^{-i\omega nt}. \tag{17}$$

The coefficients $\{\phi_{n,\alpha}\}$ are the eigenvectors of the complex-scaled Floquet Hamiltonian matrix

$$\underline{\hat{\mathcal{H}}}\Phi_\alpha = \varepsilon_\alpha\Phi_\alpha, \tag{18}$$

where

$$\hat{\mathcal{H}}_{n,m}(\mathbf{x}e^{i\theta}) = \frac{1}{T}\int_0^T \hat{H}(\mathbf{x}e^{i\theta},t)e^{i\omega(n-m)t}dt. \tag{19}$$

In the case that the time-periodic Hamiltonian is given by

$$\hat{H}(\mathbf{x}t) = \hat{H}_0(\mathbf{x}) + a\hat{H}_1(\mathbf{x})\cos(\omega t), \tag{20}$$

the diagonal $\hat{\mathcal{H}}_{n,n}$ matrix element is the field-free Hamiltonian $\hat{H}_0$ shifted by a constant that is equal to $\hbar\omega n$ $(-\infty < n < \infty)$. The exact solution given in Eqs. (17) and (18) can be expanded in a basis set constructed from the eigenfunctions $\{\psi_j^{(0)}\}$ of $\hat{\mathcal{H}}_{n,n}$ (the complex-scaled field-free Hamiltonian in our case). That is

$$\phi_\alpha = \sum_j \psi_j^{(0)}(\mathbf{x}e^{i\theta})\mathbf{C}_j^{(\alpha)}, \tag{21}$$

where the vector elements $\{C_{j,n}^{(\alpha)}\}$ are variationally obtained (for the complex-variational principle, see Ref. [14]). Of course in the absence of the electromagnetic field, i.e., $a_0=0$, the vectors $(\mathbf{C}_1^{\alpha}, \mathbf{C}_2^{(\alpha)}, \ldots, C_j^{(\alpha)})$ form a unit matrix. As the field strength is increased, the free-field states $\psi_j^{(0)}$ are coupled and in the most extreme case *extended* states are obtained when all absolute values of the variational matrix elements, $C_{nj}^{(\alpha)}$, are almost equal and differ by only a phase factor. The conditions for which such extended states are obtained are model Hamiltonian dependent and will not be discussed here. The conditions for which two specific eigenfunctions of the bare Hamiltonian are strongly coupled, however, are *not* model dependent. It is well known that the most strongly coupled states are the degenerate eigenstates of the bare Hamiltonian (i.e., "zero-order" Hamiltonian). Since quasienergies and also the eigenvalues of the diagonal Floquet Hamiltonian matrix, $\hat{\mathcal{H}}_{n,n}$, are modulo $\hbar\omega$, two degenerate eigenvalues of $\hat{\mathcal{H}}_{n,n}$ will be obtained for a laser frequency ω that satisfy the condition

$$\omega=\frac{|E_j^{(0)}-E_i^{(0)}|}{n\hbar}\ , \quad n\geq 1 \tag{22}$$

where $E_j^{(0)}$ and $E_i^{(0)}$ are two eigenvalues of the field-free Hamiltonian. When Eq. (22) is satisfied, the ith eigenvalue of the $m=0$ channel in the Floquet Hamiltonian matrix and the jth eigenvalue of the $m=-n$ channel are degenerate, whereas the jth eigenvalue of the $m=0$ channel is degenerate with the ith state of the $m=+n$ channel. As the electromagnetic field is turned on, there is a strong mixing of the field-free degenerate states resulting in a level repulsion,

$$\mathrm{Re}(\varepsilon_i)\cong E_i^{(0)}-\Delta\ ,$$

$$\mathrm{Re}(\varepsilon_j)\cong E_j^{(0)}+\Delta\ ,$$

where ε_i and ε_j are modulo $\hbar\omega$ exact almost degenerate (overlapping) resonance quasienergy levels obtained by solving Eqs. (4)–(6) or Eqs. (18) and (19). Δ is a field-strength-dependent energy splitting. For a sufficiently strong laser field it may occur that Δ is larger than the dissociation or ionization energy of the jth state of the field-free Hamiltonian. In such a case one of the two quasienergy states will be pushed into the continuum and an annihilation of one discrete quasienergy state should be observed.

Another possibility is that $E_j^{(0)}$ is a bound energy level of the bare Hamiltonian whereas

$$E_i^{(0)}=E_{\mathrm{thr}}\ . \tag{23}$$

Then the condition given in Eq. (22) is reduced to

$$\omega=|E_j^{(0)}-E_{\mathrm{thr}}|/n\hbar \tag{24}$$

and $\hbar\omega n$ is equal to the dissociation or ionization energy of the ith field-free bound state. In such a case the jth eigenstate of the $m=0$ channel of the Floquet Hamiltonian is degenerate with the threshold of the $m=-n$ channel. In one-dimensional dissociative systems the density of states is varying as $(E-E_{\mathrm{thr}})^{-1/2}$ and gets infinitely large at the threshold. Consequently, the jth bound state of the bare Hamiltonian will be strongly coupled to the scattering states as the electromagnetic field is turned on. Therefore, when Eq. (24) is satisfied (for $E_{\mathrm{thr}}=0$ as a reference energy) one may expect that

$$\lambda_j=e^{-j\varepsilon_i T/\hbar}\to 1+i0 \tag{25}$$

as the field strength a is increased, and annihilation of one discrete quasienergy state should be observed as well.

The above description of annihilation of a discrete quasienergy state by the threshold effect mechanism is the simplest one.

An example for a more complicated case is when two field-free states become degenerate as the laser field is turned on,

$$E_i^{(0)}=E_j^{(0)}+\hbar\omega m \tag{26}$$

Therefore, the zero-order quasienergy states $\phi_\pm^{(0)}$ are given by

$$\phi_\pm^{(0)}=\sqrt{\tfrac{1}{2}}(\psi_i^{(0)}\pm\psi_j^{(0)}) \tag{27}$$

and the corresponding zero-order quasienergy in the first Brillouin zone (i.e., modulo $\hbar\omega$) is given by

$$\varepsilon_\pm^{(0)}=\tfrac{1}{2}(E_i^{(0)}\pm E_j^{(0)})\ ,$$

By replacing $E_j^{(0)}$ in Eq. (2) by $\varepsilon_\pm^{(0)}$ one sees that annihilation of a discrete quasienergy state is expected when the field frequency satisfies

$$\omega=\frac{E_i^{(0)}+E_j^{(0)}}{2n\hbar} \tag{28}$$

and

$$\omega=\frac{E_i^{(0)}-E_j^{(0)}}{\hbar m}\ , \tag{29}$$

where n and m are integers and the threshold energy in the first Brillouin zone is taken as zero.

III. ILLUSTRATIVE NUMERICAL EXAMPLE

As an illustrative numerical example we studied the simple one-dimensional case where

$$\hat{H}=-\frac{\hbar^2}{2}\frac{d^2}{dx^2}+f(x)[1+a\cos(\omega t)] \tag{30}$$

and chose $f(x)$ to be the Rosen-Morse potential [15],

$$f(x)=-\frac{V_0}{\cosh^2(\alpha x)}\ , \tag{31}$$

since it does not support resonances when t is treated adiabatically, and since it has been used before as a testing ground for theories and computational methods [5,6,12,13].

In our calculations we chose $V_0=\frac{1}{2}$, $\alpha=1$, and $\hbar=1/\sqrt{240}\approx 0.065$; then $f(x)$ supports 15 bound states at the energies

$$E_n^{(0)}=-\frac{(15-n)^2}{460}\ , \quad n=0,1,2,\ldots,14\ . \tag{32}$$

The equation of motion for the complex-scaled time-evolution operator is

$$i\hbar\frac{d\mathcal{U}}{dt}=\mathcal{H}(t)\mathcal{U}\ , \tag{33}$$

where

$$\mathcal{U}(t=0)=1 \tag{34}$$

and

$$[\mathcal{H}(t)]_{ij}=\langle\psi_i^{(0)}|\hat{H}(xe^{i\theta},t)|\psi_j^{(0)}\rangle\ . \tag{35}$$

$\{\psi_j^{(0)}\}$ are the eigenfunctions of the *complex-scaled* free-field Hamiltonian.

The propagation of the initial evolution operator for one optical cycle was carried out by the Adams-Moulton predictor-corrector method [13] when $\theta=0.45$ and $\{\psi_j^{(0)}\}$ were variationally obtained by using 100 particle-in-a-box functions (box size of $A=\sqrt{209}$) as a basis set. The complex quasienergies mapped to a single Brillouin zone, $\lambda_\alpha=\exp(-i\varepsilon_\alpha T/\hbar)$, were obtained by diagonalization of $U(0|T)$ [see Eqs. (12) and (13)]. In Fig. 1 we represent the eigenvalues of U (i.e., λ_α) as the field intensity a is varied. The scattering states form a spiral as described in Sec. II. When $a=0$, then 15 bound states were obtained (real quasienergies ε_α) and $|\lambda_\alpha|=1$. In the presence of a weak field, $a=0.2$, 15 resonances were obtained. The long-lives metastable quasienergy states, $|\lambda_\alpha|\approx 1$, can assign the field-free quantum numbers, and describe the low excitation levels of the potential given in Eq. (27). As discussed in Sec. II the distance of the quasienergy solution from the origin is a visual criteria for the lifetime of the corresponding resonance state. As the distance gets smaller, the lifetime becomes shorter (zero lifetime at the origin). In a strong field, $a=0.8$, only 13 resonance quasienergy states were obtained. Two states have disap-

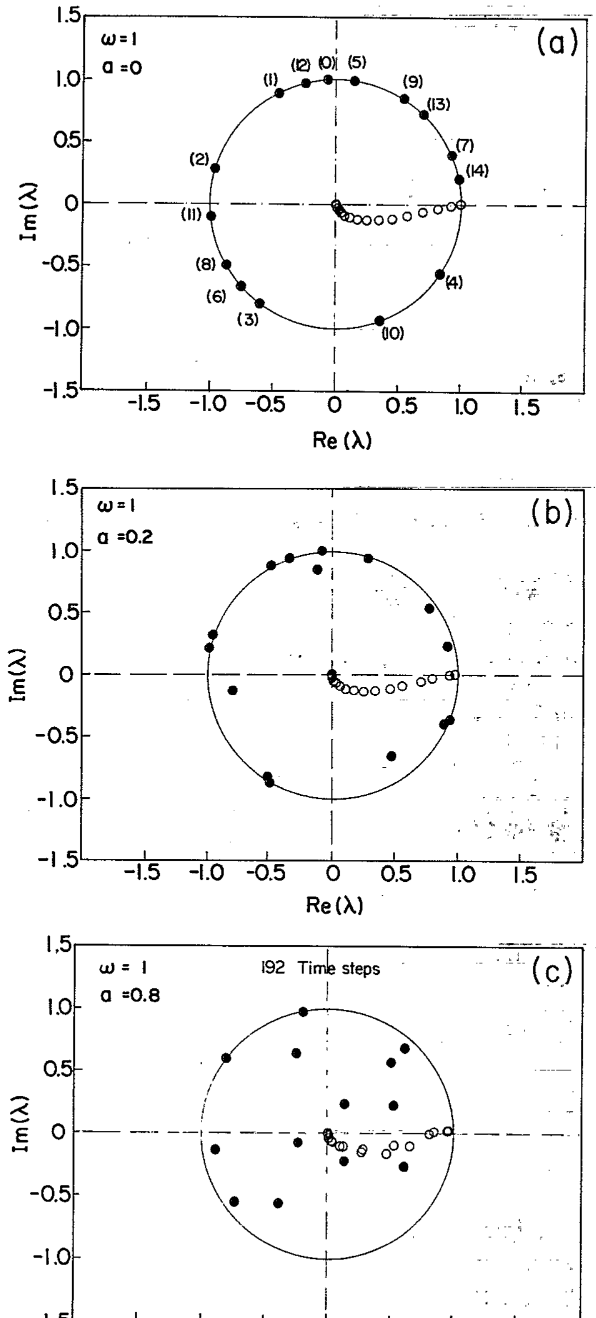

FIG. 1. Complex eigenvalues $\{\lambda i\}$ of the first optical cycle complex-scaled evolution matrix $\underline{U}(0|T)$, for the time periodic Rosen-Morse Hamiltonian [Eq. (30)]. The open circles denote the rotating continua. (a) The 15 bound states of the field-free Hamiltonian ($a=0$); (b) the 15 resonances obtained for the field intensity $a=0.2$; (c) the 13 resonances obtained for the high field intensity of $a=0.8$.

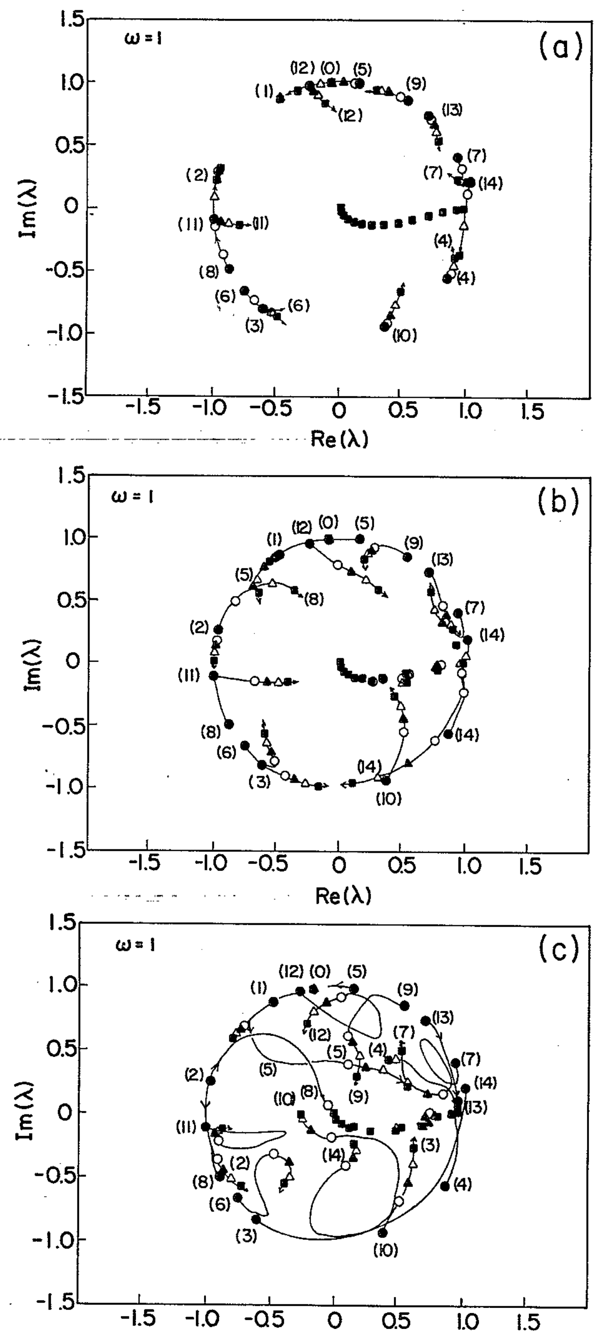

FIG. 2. The resonance complex eigenvalues of the time evolution operator after one optical cycle as the strength field intensity a is varied. (a) Resonance complex eigenvalues obtained for $a=0$, 0.05, 0.1, 0.15, and 0.2; (b) resonance eigenvalues for $a=0.25$, 0.3, 0.35, and 0.4; (c) resonance complex eigenvalues for $a=0.45$, 0.5, 0.55, and 0.6.

peared as the field intensity was increased. The mechanism of the annihilation of two discrete quasienergy states was studied by carrying out a-trajectory calculations. The results presented in Fig. 2 show the effect of increasing the field strength parameter a on the field-free states labeled 0, 1, . . . , 14. The ground and the first excited states (labeled 0 and 1) are almost unaffected by increasing a and remain very-long-lived resonances even for $a=0.8$. From Fig. 2(a) one can see that the lifetime of several states (labeled 10, 11, and 12) is reduced much more than the lifetime of the other states as the time-dependent field is turned on. Figures 2(b) and 2(c) show that by increasing the field intensity, avoided crossings of the quasienergy states labeled 6 and 10 are obtained, and also between the two quasienergy states labeled by the field-free quantum numbers 7 and 9. From Fig. 2(b) one can see state at $a=0.60$ the quasienergy state labeled by the field-free quantum number 13 is coalesced with the threshold, and then

$$\lambda_{13}(a\geq 0.6)=1+i0 . \tag{36}$$

Similarly from Fig. 2(c) one can see that the coalescence of the ninth quasienergy state with the core of the unit circle occurs at $a=0.8$ and

$$\lambda_8(a\geq 0.8)=0+i0 . \tag{37}$$

Therefore, the resonance width (inverse lifetime) is equal to

$$\Gamma_8=\infty \quad (\tau_\infty=0) . \tag{38}$$

In Sec. III we proposed two different mechanisms for the annihilation of discrete quasienergy levels. As we shall show here, the annihilation of the quasienergy level labeled by the field-free quantum number 13 results from laser-induced avoided crossings between the nine and thirteen quasienergy states, whereas the annihilation of the eight quasienergy states is due to a threshold effect.

A. Annihilation of a quasienergy state – Threshold effect

From Eq. (32) one can see that for the studied time-dependent Rosen-Morse model Hamiltonian, Eqs. (28) and (29) are satisfied for $i=6$ and $j=8$. That is,

$$\frac{E_6^{(0)}-E_8^{(0)}}{\hbar}\cong 1 \tag{39}$$

and

$$\frac{E_8^{(0)}+E_6^{(0)}}{4\hbar}\cong 1 . \tag{40}$$

From Eq. (39) we expect that by increasing the field strength (when $\omega=1$) the $j=8$ eigenstate of the field-free Hamiltonian $\hat{H}_0$ will interact with the $i=6$ eigenfunction of $\hat{H}_0$ to provide a resonance state. Indeed, from Fig. 3 one can see that for $a=0.2$ the quasienergy state labeled by the field-free quantum number 8 is a resonance state, which is constructed mainly from the $i=6$ and $j=8$ eigenfunctions of $\hat{H}_0$. From Eq. (40) we expect, on the basis of the analysis given in Sec. III, that for strong laser field the eighth quasienergy state will be pushed into the continuum and one discrete state will disappear from the spectrum (see Fig. 2). As the eighth quasienergy state is pushed to the continuum and disappears from the discrete quasienergy spectrum, it becomes an extended (continuum) state. A simple criterion for an extended state is

$$\frac{\sigma}{\hbar\omega}\gg 1 , \tag{41}$$

where

$$\sigma=|\sqrt{\langle \hat{H}_0^2\rangle-\langle \hat{H}_0\rangle^2}| . \tag{42}$$

The results presented in Fig. 4 show the variance $\sigma/\hbar\omega$ as a function of the field frequency, ω. The quasienergy states were ordered from the one that provided the smallest $\langle H_0\rangle$ value (denoted by 0) to the quasienergy state for which the largest negative value of $\langle H_0\rangle$ was obtained. A sharp transition was obtained for $\omega\approx 1$ as expected.

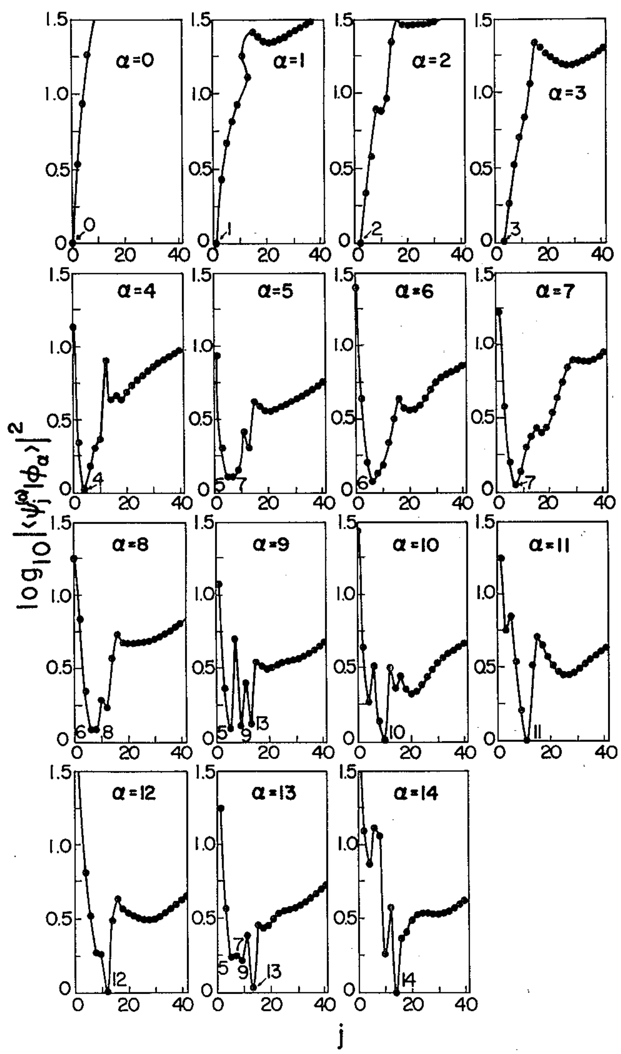

FIG. 3. The projection of the resonance quasienergy states, ϕ_α, on the eigenstates $\{\psi_j^{(0)}\}$ of the field-free Hamiltonian. $\{\phi_\alpha\}$ are the eigenfunctions of the complex-scaled time evolution matrix $\underline{U}(0|T)$ calculated for the strength field parameter $a=0$.

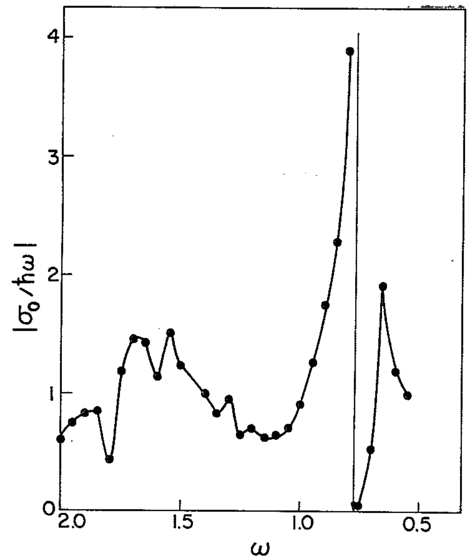

FIG. 4. The variance $\sigma/\hbar\omega$, where $\sigma=|\langle\hat{H}_0^2\rangle-\langle\hat{H}_0\rangle|^2$ as a function of the field frequency ω calculated for the quasienergy ϕ_8 ($\alpha=8$ in Fig. 2) for the strength field parameter $a=0.8$.

To check this analysis we tried to predict the field frequency for which the ground quasienergy state (i.e., the one which is associated with the minimal expectation value of $\hat{H}_0$) will disappear from the discrete quasienergy spectrum due to the threshold effect. From Eq. (24) we predict that annihilation of the ground quasienergy level will be obtained for

$$\omega=\frac{-E_0^{(0)}}{n\hbar}\simeq\frac{7.26}{n} . \tag{43}$$

Since in our numerical calculations we limited ourselves to $\omega\varepsilon[0,2]$, the critical value of the field frequency is $\omega=0.726$ when $n=10$ photons are absorbed. The results for $a=0.8$, which are presented in Fig. 4(b), confirm this analysis. Indeed, at $\omega=0.75-0.80$ a sharp transition in $\sigma/\hbar\omega$ for the ground quasienergy level was obtained and one quasienergy level disappeared from the discrete energy spectrum.

B. Annihilation of a quasienergy state–Avoided crossing mechanism (the noncrossing rule)

From Eq. (32) we see that the seven, nine, and thirteen excited bound states of the field-free Hamiltonian are almost degenerate as the time-periodic field is turned on with the frequency of $\omega=1$ ($\hbar=240^{-1/2}$),

$$E_{13}^{(0)}\simeq E_9^{(0)}+\hbar\omega\simeq E_7^{(0)}+3\hbar\omega . \tag{44}$$

These three field-free states are mixed as the field intensity is increased, as is shown in Fig. 3 (see the quasienergy labeled by quantum number 9). Since the ionization or dissociation energy of the excited field-free state 13 is very small ($E_{13}^{(0)}\approx-0.008$ a.u.), then resulting from the energy splitting of the dressed degenerate states, the dressed state 13 (also the field-free state 13 in our studied case) is pushed into the continuum and disappears from the discrete quasienergy spectrum. The dressed degenerate states are eigenfunction of the diagonal terms, $\mathcal{H}_{n,n}$, in the Floquet Hamiltonian matrix given in Eq. (19), which are reduced to $\hat{H}_0+n\hbar\omega$ in our case (where $\hat{H}_0$ is the field-free Hamiltonian). The energy splitting of the dressed degenerate states obtained as the external field is turned on results from the avoided crossing mechanism, which is also well known as the noncrossing rule [16].

IV. CONCLUDING REMARKS

The nonlinear dynamics of laser-induced atomic and molecular systems results in several striking phenomena. The suppression of slow electron peaks in the measured above-threshold ionization spectra is one example [3], and the numerical observations of creation[5–8] and drastic reduction [9] of quasienergy levels is another one [5–8]. In this work we represent numerical evidence for a physical phenomenon that is characteristic of a nonlinear dynamical process in laser-induced systems. That is, annihilation of discrete quasienergy states can be obtained, *even* when the bound states of the dressed potential are *not* affected by the field intensity.

Two different simple basic mechanisms for annihilation of states from the discrete quasienergy spectrum were described. One is based on the noncrossing rule (avoided crossing between two quasienergy levels) and the second mechanism is a threshold effect, where the ionization or dissociation energy of a dressed state is equal to $\hbar\omega n$. Of course, other mechanisms for annihilation of discrete quasienergy states that are combinations of these two basic ones are possible as well.

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