Controlled Tunneling of Cold Atoms: From Full Suppression to Strong Enhancement

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Two recent experiments have demonstrated the phenomenon of dynamical tunneling of cold atoms interacting with standing electromagnetic waves. We show by quantitative calculations that one can achieve a control of the tunneling period over an orders of magnitude range by changing the frequency difference of the waves by about 10% only. In this narrow parameter region, the mechanism of the tunneling oscillations evolves from the two-state to the three-state one. Our calculations demonstrate that the change in the underlying mechanism leads to the dramatic enhancement of the dynamical tunneling. Moreover, a complete suppression of the dynamical tunneling can also be achieved in the cold atom setup.

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The recent pioneering experiments of the Phillips [1] and Raizen [2] groups have demonstrated the dynamical tunneling of cold atoms interacting with standing electromagnetic waves. While the tunneling oscillations observed in Ref. [1] occurred by the two-state mechanism, the cold atom tunneling reported in Ref. [2] stems from the three-state process. In both cases the periods of the observed tunneling oscillations are of the same order of magnitude, despite the basic difference between the underlying mechanisms. As a result, the experiments do not afford the crucial demonstration of the control of dynamical tunneling due to the interaction with an intermediate state. We show that the physics of the dynamical tunneling can be fully explored in a cold atom experiment of the type of [1,2]. Both two-state and three-state modes of tunneling oscillations can be observed by means of a small variation of the standing wave frequencies. This variation allows one to control the tunneling period over an orders of magnitude range. Moreover, we show that the cold atom setup provides a possibility for the first observation of the full suppression of tunneling oscillations predicted theoretically in Ref. [3].

A classically forbidden transition of a particle between two regions of classically allowed motion is usually called tunneling. A standard example of this quantum mechanical effect is the motion in the so-called double-well potential. The two potential wells of this system are separated by a potential barrier which is impenetrable for a low-energy classical particle. Nevertheless, the well-known quantum solution shows that the wave packet initially localized in one of the wells performs slow oscillations between the two classically allowed regions. The period of these oscillations (or the tunneling period) is inversely related to the energy difference between the symmetric and the antisymmetric quantum states of the double-well system. Typically, the tunneling period is much larger that the period of classical oscillations inside one of the wells.

The idea of controlling the tunneling period has led many researchers to consider the effects of external forces on the tunneling oscillations. Indeed, the possibility of control of the tunneling would open up the way to a lot of new applications of this fundamental effect, e.g., in electronics and waveguide optics. Lin and Ballentine have shown that it is possible to control efficiently the tunneling in the double-well system by the use of a time-periodic external perturbation [4] (see also Ref. [5]). This effect can be seen even more clearly in the driven rotor system [6], where the tunneling causes oscillations not in the position, but rather in the angular velocity of a particle. At zero external field, no such oscillations are possible since the angular velocity is a conserved quantity. When a time-periodic external field is turned on, two stable modes of classical motion are formed in which the particle rotates in resonance with the external field either in clockwise or in counterclockwise directions. At large enough external fields, these modes are separated by a region of chaotic motion. While a classical particle trapped in one of the modes can never change its rotation direction, a quantum mechanical wave packet performs tunneling oscillations between the two modes or rotation. The phenomenon of tunneling oscillations in time-dependent systems is usually called dynamical tunneling, although the term was initially introduced to describe tunneling in a two-dimensional, time-independent potential [7].

The tunneling period in the time-dependent systems is related to the differences between quasienergies of the Floquet states, just as the tunneling period in the time-independent case has to do with the energy differences between the stationary states. While in the simplest case the period of the dynamical tunneling is given by the quasienergy splitting of a pair of odd and even Floquet states (two-state mechanism), a number of more complicated scenarios arise when one of the states undergoes interaction with a third Floquet state. In the latter case, the tunneling wave packet is described as a linear combination of the three Floquet states (three-state mechanism) [8]. The interaction of these states can drastically affect their quasienergies leading either to the degeneracy of two of them or to the dramatic increase in their quasienergy differences. In the former case, the tunneling oscillations are completely suppressed (so-called "locking"; see Ref. [3]), while the latter situation corresponds to the great reduction of the tunneling period (intermediate state enhancement of tunneling; see Ref. [9]).

Recently, the ideas of the control of tunneling by the external force have found their realization in two pioneering cold atom experiments of the Phillips [1] and Raizen [2] groups (see also the discussion in Ref. [10]). A cold atom moving in the field of two standing waves of electromagnetic radiation can be described by the driven pendulum Hamiltonian closely related to the driven rotor system of Ref. [6]. The Schrödinger equation of the driven pendulum model represented in the scaled units is [11]

$$i\hbar_{\rm eff}\frac{\partial}{\partial\tau}\Psi = \hat{H}\psi, \qquad \hat{H} = \frac{\hat{p}^2}{2} - \gamma[\theta + \cos(\tau)]\cos(q). \tag{1}$$

The three dimensionless parameters of the model are the time-independent and the general potential amplitudes, θ and γ, and the effective Planck constant, $\hbar_{\rm eff} \equiv i[\hat{p}, \hat{q}]$. The dimensionless effective Planck constant is given by $\hbar_{\rm eff} = \hbar 4k_L^2/M\delta\omega$, where k_L is the mean wave vector of the two standing waves, M is the atomic mass, and $\delta\omega$ is the frequency difference of the two standing waves. The value of $\hbar_{\rm eff}$ can be varied in an experiment by changing the frequency difference of the two standing waves, while the parameters θ and γ can be controlled also by varying the amplitudes of the waves [11]. Both of the experiments [1,2] were performed at quite large values of $\hbar_{\rm eff}$, such that the initial wave packets could not be contained in the phase space regions corresponding to the stable modes of rotation (regular islands). Consequently, the tunneling oscillations were quite fast, 10 to 20 modulation periods. Our analysis of the numerical quasienergy solutions of Eq. (1) shows that the initial wave packet in the experiment of the Phillips group was a linear combination of two (odd and even) quasienergy states localized around the stability islands. On the other hand, the initial wave packet in the experiment of the Raizen group was supported by three quasienergy states: one of them is localized in the pair of the regular islands, while the other two are localized in the regular and in the chaotic regions of the phase space. Accordingly, the tunneling oscillations in the Phillips group experiment proceed by the two-state mechanism, while the tunneling in the Raizen group experiment is achieved by the three-state mechanism. Nevertheless, one cannot see a dramatic difference in the tunneling periods in the two experiments which could be expected on the basis of the basic difference in the underlying mechanisms. This apparent discrepancy is related to the fact that the two experiments were performed at very different parameter values of the driven pendulum model and their results cannot be compared directly.

A natural question arises whether it is feasible to set up an experiment allowing one to compare the two mechanisms of tunneling. Our calculations show that such an experiment can be performed by varying the effective Planck constant of the Raizen group experiment by only about 10%. This can be achieved by the corresponding small change in the frequency difference between the two standing waves used in the experimental setup. In fact, within the narrow range of $\hbar_{\rm eff}$ one can observe (i) slow dynamical tunneling by a two-state mechanism similar to the one in Ref. [1], (ii) much faster tunneling by the three-state mechanism of Ref. [2], and (iii) full suppression of tunneling due to the field-induced degeneracy of two quasienergy states [3]. It is important to note that the latter phenomenon predicted theoretically several years ago has not yet been observed experimentally.

We obtain numerically the Floquet solutions for the Schrödinger equation (1)

$$\Psi_n = e^{-i\varepsilon_n\tau/\hbar_{\rm eff}}\Phi_n \tag{2}$$

by diagonalizing the time-evolution operator for one modulation period. Husimi distributions for the wave functions Φ_n are generated in order to classify the Floquet solutions as the ones occupying either regular or chaotic or both regions of the phase space [6]. Our calculations are performed in the interval of effective Planck constant around the value used in the Raizen group experiment ($\hbar_{\rm eff} = 2.08$). The behavior of the three Floquet states relevant for the process of dynamical tunneling in this experiment is shown in Fig. 1(a). Our calculation shows that the quasienergy states Φ_2 and Φ_3 undergo an avoided crossing around $\hbar_{\rm eff} = 2.00$, while the state Φ_1 changes more or less linearly as a function of the effective Planck constant. Outside the region of the avoided crossing (say, at $\hbar_{\rm eff} < 1.90$), the tunneling wave packet, Ψ, is described well by a linear combination of the Floquet states $\Phi_{1,2}$:

$$\Psi(\tau) = \frac{1}{\sqrt{2}}e^{-i\varepsilon_1\tau/\hbar_{\rm eff}}(\Phi_1 + e^{-i(\varepsilon_1-\varepsilon_2)\tau/\hbar_{\rm eff}}\Phi_2), \tag{3}$$

where ε_j is the quasienergy of the jth state. The difference between the quasienergies determines the period with which the wave packet oscillates between the symmetric and the antisymmetric linear combinations of the two Floquet states. These combinations correspond to left-to-right and right-to-left atomic motions, respectively. The tunneling process is detected experimentally [1,2] as the oscillations of the mean momentum of the atomic motion,

$$\langle p\rangle(\tau) = \langle\Psi(\tau)|\hat{p}|\Psi(\tau)\rangle. \tag{4}$$

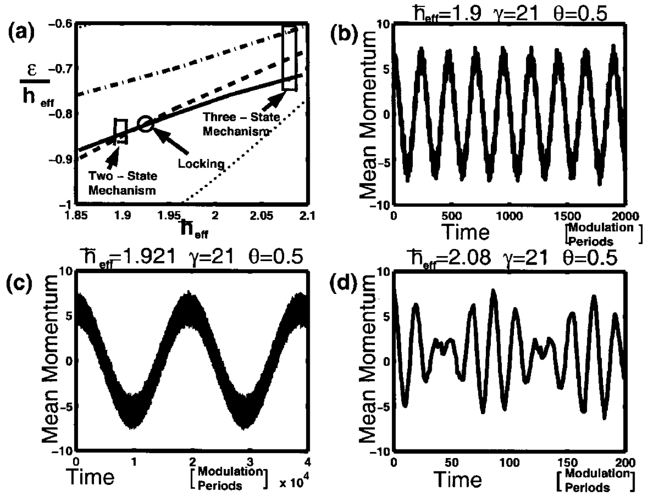

FIG. 1. (a) Quasienergies of the driven pendulum model as functions of the effective Planck constant: ε_1 (dashed line); ε_2 (solid line); ε_3 (dash-dotted line). The point of locking of tunneling, $\varepsilon_1 = \varepsilon_2$, is marked by a circle. The "three-state dynamics" box contains the three quasienergies governing the dynamics of the Raizen group experiment. The "two-state dynamics" box contains $\varepsilon_{1,2}$ leading to two-state tunneling oscillations. (b) Slow tunneling oscillations by the two-state mechanism of Eq. (3). (c) Very slow oscillations of the mean momentum in the vicinity of the crossing of the states Φ_1 and Φ_2. (d) Fast tunneling oscillations by the three-state mechanism of Eq. (5) at the parameters of the Raizen group experiment.

The period of the oscillations, i.e., the tunneling period is related to the quasienergy difference: $T = 2\pi\hbar_{\rm eff}/(\varepsilon_1 - \varepsilon_2)$. For example, at $\hbar_{\rm eff} = 1.90$ the small quasienergy splitting between the states $\Phi_{1,2}$ leads to the large tunneling period of 250 periods of the standing wave modulation. The resulting slow oscillations in the mean momentum of the cold atom are shown in Fig. 1(b).

The mixing of the states $\Phi_{2,3}$ in the region of the avoided crossing results in the change of the character of the respective wave functions. At $\hbar_{\rm eff} = 1.90$ the state Φ_2 is localized around the regular islands and the state Φ_3 is contained in the chaotic region. On the other hand, at $\hbar_{\rm eff} = 2.08$ both states possess significant probability density in both chaotic and regular regions of the phase space. Consequently, in the region of the avoided crossing the tunneling wave packet is described by a linear combination of the three Floquet states. In particular, when $\varepsilon_2 - \varepsilon_1 \approx \varepsilon_1 - \varepsilon_3 = \Delta/2$ [see the three-state dynamics box in Fig. 1(a)], one obtains

$$\Psi(\tau) = \frac{1}{\sqrt{2}} e^{-i\varepsilon_1\tau/\hbar_{\rm eff}} \times \left[\Phi_1 + \frac{1}{2}\left(e^{-i\Delta\tau/2\hbar_{\rm eff}}\Phi_2 + e^{i\Delta\tau/2\hbar_{\rm eff}}\Phi_3\right)\right]. \quad (5)$$

The period of the oscillations of the wave packet (5) between the two regular islands is given by $2\pi\hbar_{\rm eff}/\Delta$. The avoided crossing brings about a significant increase of the splitting between the quasienergies ε_1 and ε_2 [see Fig. 1(a)]. Consequently, as seen in Fig. 1(d), the period of the three-state tunneling process is much shorter than that of the two-state one: $T(\hbar_{\rm eff} = 2.08) = 20 \ll T(\hbar_{\rm eff} = 1.90)$. This strong enhancement of the tunneling is a general feature of the three-state tunneling mechanism.

The repulsion of the state Φ_2 from the state Φ_3 in the vicinity of the avoided crossing brings about the real crossing of the states Φ_1 and Φ_2 at $\hbar_{\rm eff} = 1.92$. At this value of the effective Planck constant one can approximately describe the wave packet localized in one of the regular islands as a combination of Φ_1 and Φ_2 states (3). However, zero quasienergy difference at this point leads to the infinite tunneling period, i.e., to the full suppression of the dynamical tunneling. Choosing the $\hbar_{\rm eff}$ value arbitrarily close to the crossing, one is able to generate arbitrarily slow tunneling oscillations, as illustrated in Fig. 1(c) for $\hbar_{\rm eff} = 0.921$. The effect of the dynamical tunneling suppression has been predicted by Hänggi and co-workers [3] but has never been observed experimentally.

In conclusion, the cold atom setup allows one to observe all three modes of dynamical tunneling: the slow two-state process, the strong enhancement due to the interaction with the third state, and the suppression of tunneling oscillations due to the degeneracy of the two Floquet states. This rich variety of tunneling phenomena is contained in the small interval of the effective Planck constant: $1.85 < \hbar_{\rm eff} < 2.10$. In the experiment, it is possible to vary $\hbar_{\rm eff}$ by varying the frequency difference between the two standing waves. The proposed range of the variation of this parameter lies within a 10% deviation from the value used by the Raizen group [2] and is experimentally feasible. The experiment we suggest to perform on the basis of our analysis will for the first time demonstrate the possible mechanisms of the dynamical tunneling and elucidate the qualitative and quantitative differences between them.

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