Non-evanescent adiabatic directional coupler

Alexander M. Kenis, Ilya Vorobeichik, Meir Orenstein and Nimrod Moiseyev

Abstract— **A directional coupling mechanism based on an adiabatic coupling between three optical modes is suggested. The optical power transfer between two waveguides which are far apart is mediated by adiabatic coupling between zero-order optical modes of the individual waveguides and a high-order intermediate mode. The analytical model for an adiabatic three-mode coupling based on a scalar wave equation is presented. The directional coupling via the adiabatic mode coupling between copropagating modes is described and compared with a non-adiabatic directional coupling assisted by periodic perturbation. It is shown that the adiabatic directional coupling has much less sensitivity to the mode parameters and to the wavelength.**

Keywords— **Directional couplers, multimode optical waveguides, numerical analysis, periodic structures**

I. Introduction

Directional couplers are important for many applications in optical communications and integrated optics. They can be used as power dividers, wavelength filters and for light switching and modulating [1]-[8]. In its simplest form a directional coupler consists of two parallel dielectric waveguides in a close proximity. In this case, the power coupling is based on optical interference between the evanescent modal fields of the two waveguides, such that a light wave launched into one of the waveguides can be coupled completely into the opposite guide [1], [2]. For non-synchronous waveguides, a grating assisted directional coupling (GADC) was implemented [1]-[9]. GADC is based on the direct evanescent coupling between two optical modes, which are out of phase and the grating is used for phase difference compensation. Recently, a non-evanescent directional coupling mechanism based on mode coupling in waveguides was suggested [10], [11]. In this device the power transfer between the waveguides is achieved via the coupling between each of the zero-order modes of the individual waveguides and a third common optical mode. Similar principles were subsequently used by Peral and Yariv [12] for mode conversion between optical modes of a multimode waveguide. They proposed a new type of mode converter between copropagating modes, where mode conversion is mediated by a backward-propagating mode. Adiabatic mode coupling was used to avoid the back reflection [13]. The adiabatic optical mode coupling is analogous to the coupling between quantum states used in stimulated Raman adiabatic passage (STIRAP) scheme proposed by Bergmann and co-workers [14], [15]. In this paper, we use a related formalism to suggest a directional coupling scheme based on the adiabatic mode coupling. In Section II the analytical solutions for the mode coupling based on a scalar wave equation are derived. In Section III the analytical solutions are compared with the numerically exact solutions of the scalar wave equation. The adiabatic and non-adiabatic schemes of three-mode directional coupling are compared as well. In Section IV we conclude. In the Appendix the transfer matrix method which was used to solve the scalar wave equation numerically is briefly described.

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II. Adiabatic mode coupling

For planar structures the field satisfies the scalar wave equation [2]

$$\left[\nabla^2_{x,z} + \frac{\omega^2}{c^2} n^2(x,z)\right] \Psi(x,z) = 0, \qquad (1)$$

where $\Psi(x,z)$ describes the spatial part of a scalar optical field characterized by the free space wavevector $k = 2\pi/\lambda = \omega/c$ propagating in z-direction within a slab medium which is homogeneous in y-direction and has a refractive index distribution $n(x,z)$. The waveguide is weakly guiding if the refractive index of the cladding (n_0) and that of the core (n_1) are close, i.e., $(1 - n_0^2/n_1^2) \ll 1$.

We consider a system in which the solution of the scalar wave equation is assumed to be given by a linear combination of three optical modes, such that

$$\begin{aligned}\Psi(x,z) &= C_1(z)e^{i\beta_1 z}\Phi_1^{(0)}(x) + C_2(z)e^{i\beta_2 z}\Phi_2^{(0)}(x) \\ &+ C_3(z)e^{i\beta_3 z}\Phi_3^{(0)}(x), \qquad (2)\end{aligned}$$

where $C_j(z)$ are the z-dependent coefficients of the ideal optical modes $\Phi_j^{(0)}(x)$. These ideal modes and their propagation constants, β_j, describe the optical wave propagating in a medium with z-independent refractive index $\bar{n}(x)$:

$$\left[\frac{d^2}{dx^2} + \frac{\omega^2}{c^2}\bar{n}^2(x)\right] \Phi_j^{(0)}(x) = \beta_j^2 \Phi_j^{(0)}(x). \qquad (3)$$

In a directional coupling problem, for asynchronous waveguides, $\Phi_1^{(0)}(x)$ and $\Phi_2^{(0)}(x)$ represent the optical fields of the composite structure localized in the first and the second waveguides, respectively and $\Phi_3^{(0)}(x)$ is a high-order mode of the entire structure. We consider here only the case in which the waveguides are far apart. Therefore the zero-order modes of $\Phi_1^{(0)}(x)$ and $\Phi_2^{(0)}(x)$ of the of the composite structure are very similar to modes calculated for each of the separate waveguides. In this paper the synchronous waveguide case is of no interest and thus not considered.

The refractive index profile, $\bar{n}(x)$, in the asynchronous directional coupler example is shown in Fig. 1.

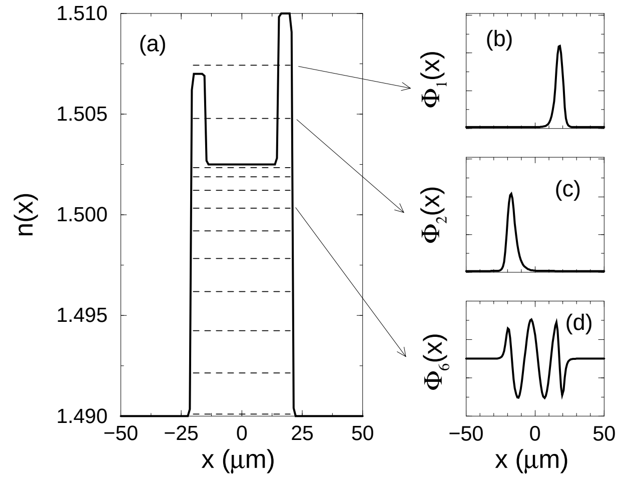

Fig. 1. (a) The refractive index distribution $\bar{n}(x)$ (solid line) and the effective indexes of the ideal modes (dashed lines). (b) Zero-order optical mode of the first waveguide. (c) Zero-order optical mode of the second waveguide. (d) A common high-order optical mode.

The three optical modes, $\Phi^{(0)}_{1,2,3}(x)$, are coupled by the modulation of the refractive index in the direction of propagation:

$$V(x,z) = \frac{\omega^2}{c^2}\left[n^2(x,z) - \bar{n}^2(x)\right]. \tag{4}$$

We assume a refractive index modulation of the form

$$\begin{aligned} V(x,z) &= \frac{\omega^2}{c^2}\left[\Delta\epsilon_1(x) g_1(z)\cos\left(\frac{2\pi}{\Lambda_1}z\right)\right. \\ &+ \left.\Delta\epsilon_2(x) g_2(z)\cos\left(\frac{2\pi}{\Lambda_2}z\right)\right], \end{aligned} \tag{5}$$

such that

$$\frac{2\pi}{\Lambda_1} = \beta_1 - \beta_3, \qquad \frac{2\pi}{\Lambda_2} = \beta_2 - \beta_3. \tag{6}$$

$\Delta\epsilon_{1,2}(x)$ represent the local amplitude of the modulation of the refractive index that couples $\Phi^{(0)}_3(x)$ with $\Phi^{(0)}_{1,2}(x)$. $g_{1,2}(z)$ are z-dependent envelope functions that represent the change in mode coupling strength in the propagation direction. The schematic representation of that model is shown in Fig. 2. The mode coupling is achieved by periodic or quasi-periodic modulation of the refractive index. For infinite periodic modulations, $g_{1,2}(z) = 1$.

We assume that the refractive index modulation is such that $\Phi^{(0)}_1(x)$ and $\Phi^{(0)}_2(x)$ are not directly coupled. In addition, the modulation of the refractive index with period Λ_1 couples between $\Phi^{(0)}_1(x)$ and $\Phi^{(0)}_3(x)$ only, whereas the second modulation with period Λ_2 couples between $\Phi^{(0)}_2(x)$ and $\Phi^{(0)}_3(x)$ only. Substituting Eq. (2) into Eq. (1) and expressing the coupling between the modes $\Phi^{(0)}_j$ in a matrix form we obtain

$$\begin{pmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \\ C_3 \end{pmatrix} =$$

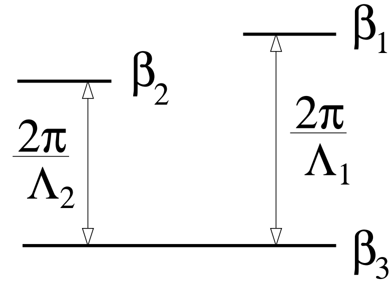

Fig. 2. The schematic representation of the three-mode optical system. The first mode (β_1) and the second mode (β_2) are coupled to the third mode (β_3) by the z-dependent modulation of the refractive index but are not coupled to each other.

$$\begin{pmatrix} 0 & 0 & V_{13}e^{-i\frac{2\pi}{\Lambda_1}z} \\ 0 & 0 & V_{23}e^{-i\frac{2\pi}{\Lambda_2}z} \\ V_{31}e^{i\frac{2\pi z}{\Lambda_1}z} & V_{32}e^{i\frac{2\pi z}{\Lambda_2}z} & 0 \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \\ C_3 \end{pmatrix}, \tag{7}$$

where $d_{1,2,3} \equiv \frac{d^2}{dz^2} + 2i\beta_{1,2,3}\frac{d}{dz}$. The z-dependent coupling matrix elements are given by

$$V_{13}(z) = V^*_{31}(z) = \kappa_1(z)\cos\left(\frac{2\pi}{\Lambda_1}z\right) \tag{8}$$

and

$$V_{23}(z) = V^*_{32}(z) = \kappa_2(z)\cos\left(\frac{2\pi}{\Lambda_2}z\right), \tag{9}$$

where $\kappa_{1,2}(z)$ are the z-dependent coupling coefficients given by z-independent constant coupling coefficients $\kappa^{(0)}_{1,2}$ multiplied by z-dependent envelopes $g_{1,2}(z)$:

$$\kappa_{1,2}(z) = g_{1,2}(z)\kappa^{(0)}_{1,2}, \tag{10}$$

where

$$\kappa^{(0)}_1 = \frac{\omega^2}{c^2}\int_{-\infty}^{\infty}\Phi^{(0)}_1(x)\Delta\epsilon_1(x)\Phi^{(0)}_3(x)dx \tag{11}$$

and

$$\kappa^{(0)}_2 = \frac{\omega^2}{c^2}\int_{-\infty}^{\infty}\Phi^{(0)}_2(x)\Delta\epsilon_2(x)\Phi^{(0)}_3(x)dx. \tag{12}$$

For small coupling coefficients the rate of power flow between the optical modes is small (within a propagation distance comparable to a single optical wavelength) and, therefore, the coefficients, $C_j(z)$, are slowly varying functions, such that

$$\left|\frac{d^2C_j(z)}{dz^2}\right| \ll \left|2i\beta_j\frac{dC_j(z)}{dz}\right|. \tag{13}$$

With Eq. (13) and using Eqs. (8,9), Eq. (7) transforms to

$$4i\frac{d}{dz}\begin{pmatrix} C_1 \\ C_2 \\ C_3 \end{pmatrix} =$$

$$\begin{pmatrix} 0 & 0 & \kappa_1 y_1/\beta_1 \\ 0 & 0 & \kappa_2 y_2/\beta_2 \\ \kappa_1 y^*_1/\beta_3 & \kappa_2 y^*_2/\beta_3 & 0 \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \\ C_3 \end{pmatrix}, \tag{14}$$

where

$$y_{1,2}(z) = 1 + e^{-2i\pi z/\Lambda_{1,2}}. \quad (15)$$

Further we assume that the variations of $\kappa_1(z)$ and $\kappa_2(z)$ are much slower than that of $y_{1,2}(z)$. The mode coupling occurs over distances much longer than the periods of the refractive index modulations (at least few tens of $\Lambda_{1,2}$). Therefore, we may replace the rapid variations of $y_{1,2}(z)$ with its average over one period of refractive index variations, i.e.,

$$y_{1,2}(z) \simeq 1. \quad (16)$$

Using this approximation (which is analogous to the rotating wave approximation in quantum mechanics), Eq. (14) is simplified:

$$4i\frac{d}{dz}\begin{pmatrix} C_1 \\ C_2 \\ C_3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & \kappa_1/\beta_1 \\ 0 & 0 & \kappa_2/\beta_2 \\ \kappa_1/\beta_3 & \kappa_2/\beta_3 & 0 \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \\ C_3 \end{pmatrix}. \quad (17)$$

Note that Eq. (17) does not depend on the direction of propagation of the modes, i.e., $\beta_{1,2,3}$ can be either positive or negative. In the case of counter-propagating modes, the required period of the refractive index modulations is in the sub-wavelength range. Eq. (17) can be solved exactly to yield the eigenvectors and eigenvalues. However, it is non-Hermitian and in order to simplify the presentation we reduce Eq. (17) to the case where all three modes are co-propagating (i.e., using long period gratings), $(\beta_1 - \beta_3)/k_0 \ll 1$ and $(\beta_2 - \beta_3)/k_0 \ll 1$ (which is referred to a paraxial approximation), where k_0 is the wavevector in the cladding (or in the core). In this case (which is relevant to the actual waveguide setting) we can assume $\beta_1 \simeq \beta_2 \simeq \beta_3 \simeq k_0$ (only for the calculation of coefficients C_i). Thus the matrix on the right hand side is Hermitian and the derivation becomes much more transparent:

$$4ik_0\frac{d}{dz}\begin{pmatrix} C_1 \\ C_2 \\ C_3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & \kappa_1 \\ 0 & 0 & \kappa_2 \\ \kappa_1 & \kappa_2 & 0 \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \\ C_3 \end{pmatrix}. \quad (18)$$

We can rewrite Eq. (18) as

$$4ik_0\frac{d}{dz}\vec{C}(z) = \mathbf{A}(z)\vec{C}(z), \quad (19)$$

where $\mathbf{A}(z)$ is a 3x3 matrix in Eq. (18). The solution of Eq. (19) can be written in a general form as

$$\vec{C}(z) = \sum_j a_j(z)\vec{D}_j(z)e^{(-i/4k_0)\int_0^z \zeta_j(z')dz'}, \quad (20)$$

where $a_j(z)$ are the coefficients of weakly z-dependent vectors $\vec{D}_j(z)$ and ζ_j are corresponding phase factors. For sufficiently slow power flow between the optical modes, one can assume that the coefficients $a_j(z)$ are z-independent and are given by the initial modal distribution of optical power at $z = 0$:

$$a_j(z) \simeq a_j = C_j(z = 0). \quad (21)$$

Assuming that initially only one optical mode is excited and substituting Eq. (20) into Eq. (19) we obtain

$$\left[-4ik_0\frac{d}{dz} + \mathbf{A}(z)\right]\vec{D}(z) = \zeta(z)\vec{D}(z). \quad (22)$$

If $\vec{D}(z)$ are sufficiently slowly (adiabatically) varying functions, then, at every z,

$$\left|\vec{D}_i\left(4ik_0\frac{d}{dz}\vec{D}_j\right)\right| \ll |\zeta_i - \zeta_j|. \quad (23)$$

In this case $\vec{D}(z)$ are the eigenvectors of the matrix $\mathbf{A}(z)$, and the z-dependence of $\vec{D}_j$ is only parametric:

$$\mathbf{A}(z)\vec{D}_j(z) = \zeta_j(z)\vec{D}_j(z). \quad (24)$$

In other words, the $\vec{D}_j(z)$ vectors adiabatically follow the z-dependence of $\mathbf{A}(z)$.

The eigen-values and the eigen-vectors of the matrix $A(z)$ can be obtained analytically. If we introduce the unit vectors

$$\vec{1} \equiv \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix};\ \vec{2} \equiv \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix};\ \vec{3} \equiv \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad (25)$$

the normalized eigenvectors and corresponding eigenvalues of the 3x3 matrix A in Eq. (18) are given by

$$\begin{aligned}
\vec{D}_0(z) &= \frac{\kappa_2}{\sqrt{\kappa_1^2+\kappa_2^2}}\vec{1} - \frac{\kappa_1}{\sqrt{\kappa_1^2+\kappa_2^2}}\vec{2} \\
\vec{D}_-(z) &= \frac{1}{\sqrt{2}}\left(\frac{\kappa_1}{\sqrt{\kappa_1^2+\kappa_2^2}}\vec{1} + \frac{\kappa_2}{\sqrt{\kappa_1^2+\kappa_2^2}}\vec{2} - \vec{3}\right) \quad (26) \\
\vec{D}_+(z) &= \frac{1}{\sqrt{2}}\left(\frac{\kappa_1}{\sqrt{\kappa_1^2+\kappa_2^2}}\vec{1} + \frac{\kappa_2}{\sqrt{\kappa_1^2+\kappa_2^2}}\vec{2} + \vec{3}\right);
\end{aligned}$$

$$\begin{aligned}
\zeta_0(z) &= 0 \\
\zeta_-(z) &= -\sqrt{\kappa_1^2+\kappa_2^2} \quad (27) \\
\zeta_+(z) &= +\sqrt{\kappa_1^2+\kappa_2^2}.
\end{aligned}$$

The z-dependence of the coefficients $D(z)$ is given by the form of $\kappa_1(z)$ and $\kappa_2(z)$. If we choose $\kappa_1(z)$ and $\kappa_2(z)$ such that initially

$$\kappa_1(z=0) \ll \kappa_2(z=0) \quad (28)$$

we obtain that

$$\frac{\kappa_1}{\sqrt{\kappa_1^2+\kappa_2^2}} \simeq 0;\ \frac{\kappa_2}{\sqrt{\kappa_1^2+\kappa_2^2}} \simeq 1. \quad (29)$$

Therefore,

$$\begin{aligned}
\vec{D}_0(0) &= \vec{1}, \\
\vec{D}_-(0) &= \frac{1}{\sqrt{2}}\vec{2} - \frac{1}{\sqrt{2}}\vec{3}, \quad (30) \\
\vec{D}_+(0) &= \frac{1}{\sqrt{2}}\vec{2} + \frac{1}{\sqrt{2}}\vec{3}.
\end{aligned}$$

Consequently, if initially (at $z = 0$) the optical power is carried by a first mode $\Phi_1^{(0)}(x)$, i.e., $\vec{C}(0) = \vec{1}$, and the condition (23) is fulfilled, we obtain from Eqs. (20,21,27) that the solution of Eq. (19) is given by the first adiabatic vector $D_0(z)$:

$$\vec{C}(z) = \vec{D}_0(z). \tag{31}$$

Therefore,

$$\begin{aligned} C_1(z) &= \frac{\kappa_2(z)}{\sqrt{\kappa_1^2(z)+\kappa_2^2(z)}}, \\ C_2(z) &= -\frac{\kappa_1(z)}{\sqrt{\kappa_1^2(z)+\kappa_2^2(z)}}, \\ C_3(z) &= 0 \end{aligned} \tag{32}$$

and

$$\Psi(x,z) = \frac{\kappa_2}{\sqrt{\kappa_1^2+\kappa_2^2}}\Phi_1^{(0)}(x)e^{i\beta_1 z} - \frac{\kappa_1}{\sqrt{\kappa_1^2+\kappa_2^2}}\Phi_2^{(0)}(x)e^{i\beta_2 z}. \tag{33}$$

If we choose $\kappa_1(z)$ and $\kappa_2(z)$ such that

$$\kappa_1(z = L) \gg \kappa_2(z = L) \tag{34}$$

we obtain

$$|C_2(z = L)|^2 = 1. \tag{35}$$

Thus, the optical power initially carried by the optical mode $\Phi_1^{(0)}(x)$ is transferred completely to the optical mode $\Phi_2^{(0)}(x)$. Note that $\Phi_1^{(0)}(x)$ and $\Phi_2^{(0)}(x)$ are not exchanging power directly and each of them is coupled to the third mode $\Phi_3^{(0)}(x)$ by $\kappa_1(z)$ and $\kappa_2(z)$, respectively. However, from Eq. (33) one can see that the third mode does not carry optical power in all the range of propagation, provided the adiabatic condition (23) is fulfilled. Moreover, the adiabatic power transfer is obtained by a sequence of couplings $\kappa_1(z)$ and $\kappa_2(z)$ (see Eq. (28) and Eq. (34)). That is, in order to transfer the optical power from the first to the second mode without exciting the auxiliary (third) mode, one needs to pre-open the draining channel from the third mode to the second (κ_2), and only than to switch on the coupling between the first and the third optical modes (κ_1). Furthermore, the second mode becomes gradually excited, the coupling κ_2 should be gradually switched off in order to prevent the power flowing in the back direction. Both the adiabaticity of the mode coupling and the efficiency of the optical power transfer depend on the form of the mode coupling coefficients $\kappa_1(z)$ and $\kappa_2(z)$. By examining Eq. (28), Eq. (34) and Eq. (33) one can see that the z-dependent envelops of the coupling coefficients need to overlap since the mode coupling strength between $\Phi_1^{(0)}(x)$ and $\Phi_3^{(0)}(x)$ increases with z while the mode coupling strength between $\Phi_2^{(0)}(x)$ and $\Phi_3^{(0)}(x)$ decreases with z. The precise z-dependence of the coupling strength envelopes is not important, however, $\kappa_{1,2}(z)$ should be sufficiently slowly varying functions to maintain the adiabatic power transfer.

In order to obtain the adiabaticity condition we substitute $\vec{D}_j(z)$ from Eqs. (26) into Eq. (23). We obtain

$$k_0 \left|\frac{d\theta}{dz}\right| \ll \sqrt{\kappa_1^2+\kappa_2^2}, \tag{36}$$

where θ is defined as

$$\tan\theta(z) = \frac{\kappa_1(z)}{\kappa_2(z)}. \tag{37}$$

Thus, the adiabaticity parameter is given by

$$\alpha(z) = \frac{k_0\,|d\theta/dz|}{\sqrt{\kappa_1^2+\kappa_2^2}}. \tag{38}$$

If the coupling coefficients $\kappa_1(z)$ and $\kappa_2(z)$ satisfy $\alpha(z) \ll 1$ and a coupling between the second and the third mode precedes the coupling between the first and the third mode, the optical power is transferred from the first to the second mode, and the third mode never carries optical power. The third mode serves as a mediator between directly uncoupled first and second modes. The adiabaticity of the mode coupling is obtained by properly changing the mode coupling strength along the propagation direction.

The example of $\kappa_1(z)$ and $\kappa_2(z)$ and the adiabaticity parameter are shown in Fig. 3.

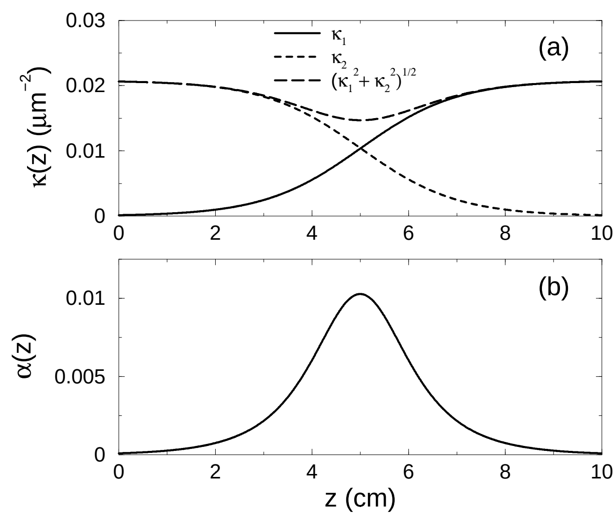

Fig. 3. (a) An example of the z-dependent coupling coefficients as defined in Eqs. (10,11,12). (b) The corresponding adiabaticity parameter, $\alpha(z)$, as defined in Eq. (38).

In a simpler case of constant mode coupling strength, the modal power flow is periodic [10], [11]. For constant and equal mode coupling coefficients, i.e., $\kappa_{1,2}(z) = \kappa_1^{(0)} = \kappa_2^{(0)} \equiv \kappa^{(0)}$, the eigen-vectors and the eigen-values of the 3x3 matrix in Eq. (18) are given by

$$\begin{aligned} \vec{D}_0 &= \frac{1}{\sqrt{2}}\vec{1} - \frac{1}{\sqrt{2}}\vec{2}, & \zeta_0 &= 0 \\ \vec{D}_- &= \frac{1}{2}\vec{1} + \frac{1}{2}\vec{2} - \frac{1}{\sqrt{2}}\vec{3}, & \zeta_- &= -\sqrt{2}\kappa^{(0)} \\ \vec{D}_+ &= \frac{1}{2}\vec{1} + \frac{1}{2}\vec{2} + \frac{1}{\sqrt{2}}\vec{3}, & \zeta_+ &= \sqrt{2}\kappa^{(0)}. \end{aligned} \tag{39}$$

Since, in a constant mode-coupling case, the 3x3 matrix in Eq. (18) is z-independent, the solution of Eq. (18) is given by a linear combination of the eigen-vectors:

$$\vec{C}(z) = \sum_j a_j \vec{D}_j e^{(-i/4k_0)\zeta_j z}, \tag{40}$$

where a_j are given by the initial mode distribution $\vec{C}(z = 0)$. If we assume that at $z = 0$ only the first mode is excited (i.e., $\vec{C}(z = 0) = \vec{1}$) and use Eqs. (39) we obtain

$$\vec{C}(z) = \frac{1}{\sqrt{2}}\vec{D}_0 + \frac{1}{2}\vec{D}_- e^{i\kappa z} + \frac{1}{2}\vec{D}_+ e^{-i\kappa z}, \tag{41}$$

where

$$\kappa = \frac{\kappa^{(0)}}{2\sqrt{2}k_0} \tag{42}$$

and $\kappa^{(0)}$ is defined in Eq. (11). Substitution of Eqs. (39) into Eq. (41) leads to

$$\begin{aligned} C_1(z) &= \frac{1}{2}\cos(\kappa z) + \frac{1}{2} \\ C_2(z) &= \frac{1}{2}\cos(\kappa z) - \frac{1}{2} \\ C_3(z) &= -\frac{i}{\sqrt{2}}\sin(\kappa z). \end{aligned} \tag{43}$$

As one can see from Eqs. (43), the optical power, initially carried by the first optical mode, is transferred completely to the second optical mode at $z_b = \pi/\kappa$, via the third intermediate optical mode, which carries 50% of the optical power at $z_c = z_b/2$. Comparing the adiabatic coefficients of the three optical modes (Eqs. (32)) with the non-adiabatic ones (Eqs. (43)), one can see that in the adiabatic coupling case, the optical power is transferred between two modes via the third mode, such that the third mode does not carry any optical power. In the non-adiabatic constant coupling case, the optical power oscillates between all three modes, such that at the coupling distance, total power is transferred between the first and the second mode.

III. Directional coupler via adiabatic mode coupling

In a previous work [10], [11] we illustrated that the directional coupling between the modal fields of two waveguides which are far apart can be achieved by using an intermediate optical mode. In this case, the refractive index distribution is such that, in addition to the zero-order modes of each of the waveguides, there are extended optical modes that are common to both waveguides. An example of such a structure is shown in Fig. 1(a). As one can see in Fig. 1(a), the refractive index in the region between the two waveguides ($n = 1.5025$) is higher than the refractive index of the cladding ($n = 1.49$), thus, supporting extended modes common to both waveguides. One of these modes is shown in Fig. 1(d). Refractive indexes of the waveguides are $n = 1.507$ and $n = 1.51$ and their width is $6\mu m$. Since the waveguides are far apart ($30\ \mu m$, in our studied case), the direct evanescent coupling between the zero-order modes of the waveguides (shown in Fig. 1(b,c)) is negligible. However, both zero-order modes can be coupled to a third high-order mode by a periodic or quasi-periodic modulation of the refractive index.

In our illustrative numerical example we use a modulation defined in Eq.(5), with a step–like function instead of a cosine function in the z direction. The perturbation in the x direction is given by

$$\Delta\epsilon_{1,2} = \alpha(1 \pm \tanh(b(x \mp x_a))) \tag{44}$$

where $\alpha = 0.0005$, $b = \ln(0.9/0.1)/5\mu m^{-1}$, and $x_a = 18.5\mu m$. The wavelength is $1.5\mu m$. The modulation periods ($\Lambda_{1,2}$) are $347.8\mu m$ and $218.2\mu m$.

If the zero-order modes of each waveguide are simultaneously coupled to the third mode ($g_1(z) = g_2(z) = 1$), the optical power initially carried by the zero-order mode of one waveguide is transferred to the zero-order mode of another waveguide through the third (intermediate) high-order mode. The coefficients of the three modes coupled by two periodic couplings are shown in Fig 4(a). The results are obtained using the transfer matrix technique, described in the Appendix. As one can see, for a constant mode coupling, the optical power oscillates between three modes such that the power is transferred from the first mode to the second via the third mode which carries 50% of the optical power at half the transfer length (Eqs. (43)).

In the case of the adiabatic mode coupling a different type of power transfer is obtained. Using the quasi-periodic adiabatic coupling as defined in Eqs. (10,28,34) the optical power is transferred between the waveguides via the adiabatic coupling to the intermediate mode without transferring net optical power to this mode. The example of the mode coefficients in this case are as shown in Fig 4(b). As one can see, the power transfer between the waveguides is achieved and the third intermediate mode carries only very small portion of the optical power. The amount of optical power carried by the third intermediate mode goes to zero as the adiabaticity parameter become smaller (Eq. (32) and Eq. (38)). The envelopes of the refractive index modulations and the adiabaticity parameter corresponding to the results shown in Fig. 4 are shown in Fig. 3. Since the adiabaticity parameter is small, a good correspondence between the exact solutions of the scalar wave equation and the three-mode approximation described in the previous section is obtained. However, the adiabaticity parameter is not small enough to entirely avoid the optical power transfer to the intermediate mode.

For the adiabaticity condition to be fulfilled the mode coupling coefficients should vary slowly with z. Therefore, the adiabatic power transfer occurs over distances that are longer than in the periodic non-adiabatic coupling case. The transfer distance can be reduced if larger mode coupling coefficients are used. The simplest way to increase the mode coupling is to increase the amplitude of the refractive index modulation. However, if the refractive index modulation is too strong additional high-order modes contribute and the three-mode approximation is not valid. Therefore, the power transfer efficiency between the waveguides

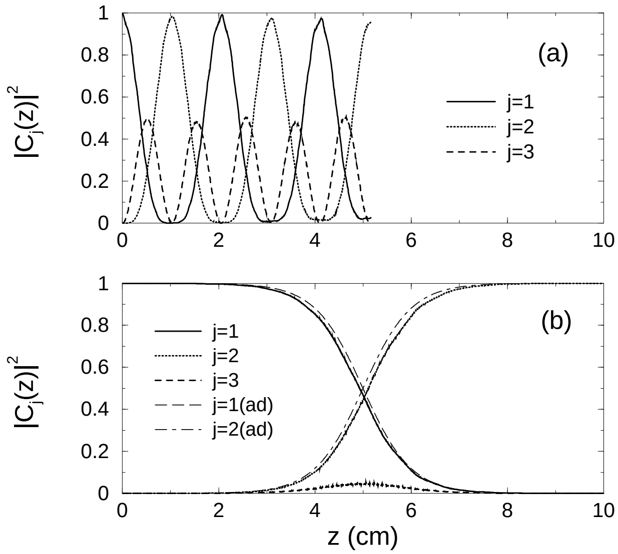

Fig. 4. (a) The z-dependent coefficients of the three optical modes coupled by two periodic modulations of the refractive index ($g_{1,2} = 1$ in Eq. (5)). Solid line stands for the initially excited zero-order mode of the first waveguide, dotted line stands for the zero-order mode of the second waveguide and the dashed line stands for the common high-order mode. (b) The z-dependent coefficients of the three optical modes coupled by two quasi-periodic modulations of the refractive index. Solid, dotted and dashed lines are as in Fig 4(a). Long dashed and dot-dashed lines stand for the analytical adiabatic coefficients of the zero-order modes of two waveguides as obtained from the three-mode adiabatic coupling model (Eqs. (32)).

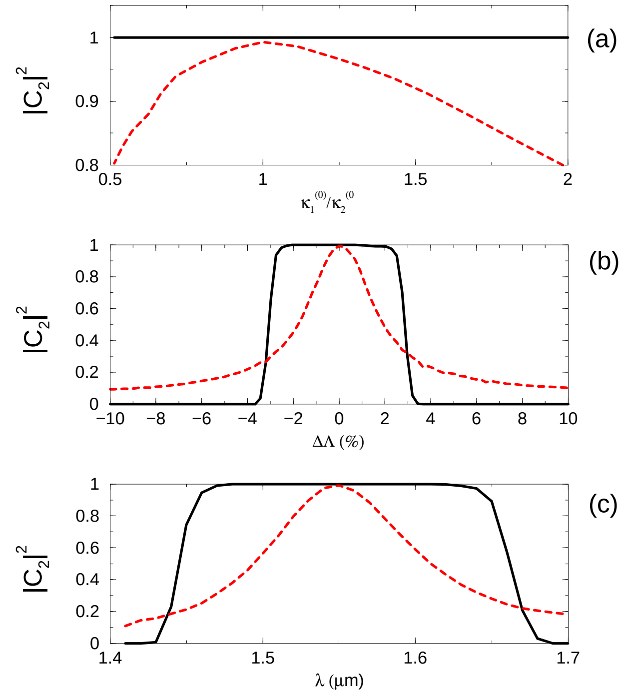

Fig. 5. (a) The optical power transferred to the zero-order mode of the second waveguide for different ratios of mode coupling coefficients defined in Eqs. (11,12). Solid line stands for the quasi-periodic adiabatic coupling and dashed line stands for the periodic non-adiabatic coupling case.
(b) The optical power transferred to the zero-order mode of the second waveguide as a function of the modulation period mismatch between the first and the third optical modes:
$\Delta\Lambda = \left(\frac{\Lambda}{2\pi/(\beta_1-\beta_3)} - 1\right) \times 100\%$.
(c) The optical power transferred to the zero-order mode of the second waveguide as a function of the wavelength.

is reduced because of the modal noise. The non-adiabatic coupling between copropagating modes can also result in shorter power transfer distances because there is no need to change the mode coupling strength slowly. However, in a non-adiabatic mode coupling case the maximal mode coupling strength for which the three-mode approximation is still valid, is, typically, smaller than in the adiabatic coupling case. Therefore, although the adiabatic mode coupling requires slow change of the mode-coupling strength, the maximal coupling strength can be high. (In our example, the coupling coefficients corresponding to Fig. 4(b) are five times larger than those used in Fig. 4(a)). Another possible way to reduce the transfer distance is to employ a coupling between the counter propagating modes [5]. In this case, the coupling distance is smaller, however, subwavelength gratings are needed and a total suppression of reflections is difficult to achieve. In spite of the longer transfer distance, the adiabatic mode coupling between copropagating modes has several advantages, the most important of which is the robustness of the coupling. If Fig. 5 the dependence of the power transfer between waveguides on different parameters is illustrated.

In Fig. 5(a) the power transfer between waveguides for different coupling ratios is presented. For non-adiabatic coupling case the power transfer is most efficient for equal coupling strength between the zero-order modes of the waveguides and the intermediate mode. For different mode couplings the transfer efficiency is reduced. The adiabatic mode coupling, however, does not require equal couplings and is efficient for wide range of mode coupling ratios. Since the adiabatic power transfer has very low sensitivity to the ratio between coupling strengths, it can be used with the same efficiency for both synchronous and asynchronous directional couplers. For asynchronous directional couplers, the two waveguides are different and, therefore, if the same perturbation is used, the mode coupling strength between the zero-order mode of each waveguide and the intermediate mode is different. In this case, the efficiency of the non-adiabatic power transfer will be reduced. However, the efficiency of the adiabatic power transfer will not be affected by the difference between the waveguides. Similar result is obtained if the adiabatic and non-adiabatic transfer are compared as a function of a mismatch between the propagation constants and the period of the refractive index modulation. In Fig. 5(b) the power transfer for different shifts in the modulation period between the zero-order mode of the first waveguide and the third mode is shown. As one can see, the non-adiabatic power transfer reduces very fast with the period mismatch. The adiabatic power transfer, however, is unchanged even if the modulation period is 3% larger (or smaller) than in the resonant case.

In Fig. 5(c) the wavelength dependence of the directional coupling is shown. As one can see, the wavelength dependence of both adiabatic and non-adiabatic directional coupling schemes is weak because the coupling between co-propagating modes is used. In the adiabatic coupling case, the power transfer is very close to 100% in a very wide range of wavelengths. The effect of low wavelength sensitivity of the adiabatic power transfer is related to the low sensitivity of the adiabatic power transfer to the mode coupling strength and to the modulation period mismatch. Both the modulation period and the mode coupling strength change when the wavelength is modified. However, since the adiabatic power transfer has weak dependence on these parameters it also has low sensitivity to the wavelength variations. The wavelength-insensitivity of the adiabatic power transfer can be very important for various integrated optics applications.

IV. Conclusions

The directional coupling mechanism based on an adiabatic coupling between three optical modes is suggested. The optical power transfer between two waveguides is demonstrated, although the waveguides are far apart and the direct evanescent coupling between zero-order modal fields is negligible. The optical power transfer is achieved by adiabatic coupling between zero-order optical modes and a high-order intermediate mode. Due to the adiabatic nature of the coupling the intermediate mode carries only very small portion of optical power. The exact solutions of the scalar wave equation are in a good agreement with the analytical results obtained from a three-mode coupling model. The directional coupling via the adiabatic mode coupling between copropagating modes is compared with the non-adiabatic periodic mode coupling. It is shown that adiabatic directional coupling has very small sensitivity to the variation of the mode coupling parameters and the wavelength.

V. Appendix : The transfer matrix method

The scalar wave equation was solved using the transfer matrix method, which is briefly described below. It is based on dividing the non-uniform refractive index distribution into a regions with constant refractive index, for which the solutions of the wave equations are known. The solution of the wave equation is obtained by matching the solutions in neighboring regions.

We use the transfer matrix formalism to calculate the transmission and reflection coefficients of transverse optical modes which are coupled by a z-dependent non-uniformity of the refractive index [16]. We consider the scalar wave (Helmholtz) equation

$$\left[\nabla^2_{x,y,z} + \frac{\omega^2}{c^2} n^2(x,y,z)\right] \Psi(x,y,z) = 0 \qquad (45)$$

and assume $n(x,y,z) = n_j(x,y)$ in each region j. Therefore, in each separate region

$$\Psi^{(j)}(x,y,z) = \Phi^{(j)}(x,y) e^{i\beta(j)z}, \qquad (46)$$

where

$$\left[\nabla^2_{x,y} + \frac{\omega^2}{c^2} n_j^2(x,y)\right] \Phi^{(j)}(x,y) = \beta^2(j)\Phi^{(j)}(x,y). \qquad (47)$$

A general solution in each region in which the refractive index is assumed to be z-independent is given by a linear combination of optical modes:

$$\begin{aligned}\Psi^{(j)}(x,y,z) &= \sum_{l=1}^{N} t_l^{(j)} \Phi_l^{(j)}(x,y) e^{i\beta_l(j)z} \\ &+ \sum_{l=1}^{N} r_l^{(j)} \Phi_l^{(j)}(x,y) e^{-i\beta_l(j)z}, \qquad (48)\end{aligned}$$

where $t_l^{(j)}$ and $r_l^{(j)}$ are the unknown coefficients of forward- and backward-propagating modes in the region j, and N is the total number of modes chosen to describe the solution of the wave equation. Therefore, the continuity condition between the solution in region j and in the adjacent region $j+1$ is given by

$$\begin{aligned}\Psi^{(j)}(x,y,z=Z_j) &= \Psi^{(j+1)}(x,y,z=Z_j) \qquad (49)\\ \frac{\partial}{\partial z}\Psi^{(j)}(x,y,z=Z_j) &= \frac{\partial}{\partial z}\Psi^{(j+1)}(x,y,z=Z_j),\end{aligned}$$

where Z_j is the boundary between region j and $j+1$.

To write Eqs. (49) in a matrix form we need to express $\Phi_l^{(j)}(x,y)$ and $\Phi_l^{(j+1)}(x,y)$ in the same basis of ideal modes $\Phi_m^{(0)}(x,y)$:

$$\Phi_l^{(j)}(x,y) = \sum_{m=1}^{N} A_{m,l}^{(j)} \Phi_m^{(0)}(x,y), \qquad (50)$$

where

$$\left[\nabla^2_{x,y} + \frac{\omega^2}{c^2} \bar{n}^2(x,y)\right] \Phi_m^{(0)}(x,y) = \bar{\beta}_m^2 \Phi_m^{(0)}(x,y), \qquad (51)$$

where $\bar{n}(x,y)$ is the reference z-independent refractive index distribution. The coefficients of the ideal modes in each region are obtained from

$$\left(\left[\bar{\beta}^2\right]^d + \mathbf{V}^{(j)}\right)\mathbf{A}^{(j)} = \mathbf{A}^{(j)}\left[\beta^2(j)\right]^d, \qquad (52)$$

where

$$\begin{aligned}V_{m,l}^{(j)} &= \frac{\omega^2}{c^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dxdy\ \Phi_m^{(0)}(x,y) \\ &\quad \left[n_j^2(x,y) - \bar{n}^2(x,y)\right]\Phi_l^{(0)}(x,y). \qquad (53)\end{aligned}$$

$\left[\bar{\beta}^2\right]^d$ and $\left[\beta^2(j)\right]^d$ are diagonal matrices with $\bar{\beta}_m^2$ and $\beta_l^2(j)$ on the diagonal. Using the same basis of modes in each region we can rewrite Eqs. (49) in a matrix form:

$$\mathbf{U}^{(j)}\mathbf{E}^{(j)}\begin{pmatrix} t^{(j)} \\ r^{(j)} \end{pmatrix} = \mathbf{U}^{(j+1)}\mathbf{E}^{(j+1)}\begin{pmatrix} t^{(j+1)} \\ r^{(j+1)} \end{pmatrix}, \qquad (54)$$

where $\mathbf{U}^{(j)}$ and $\mathbf{E}^{(j)}$ are 2Nx2N matrices given by

$$\mathbf{U}^{(j)} = \begin{pmatrix} \mathbf{A}^{(j)} & 0 \\ 0 & \mathbf{A}^{(j)} \end{pmatrix} \tag{55}$$

and

$$\mathbf{E}^{(j)} = \begin{pmatrix} \mathbf{E}_+^{(j)} & \mathbf{E}_-^{(j)} \\ \mathbf{E}_+^{\prime(j)} & \mathbf{E}_-^{\prime(j)} \end{pmatrix}. \tag{56}$$

The matrix elements of $\mathbf{A}^{(j)}$ are given by Eq. (52) and $\mathbf{E}_+^{(j)}$, $\mathbf{E}_-^{(j)}$, $\mathbf{E}_+^{\prime(j)}$ and $\mathbf{E}_-^{\prime(j)}$ are diagonal matrices with $N_l e^{i\beta_l(j)z}$, $N_l e^{-i\beta_l(j)z}$, $N_l i\beta_l(j)e^{i\beta_l(j)z}$ and $-N_l i\beta_l(j)e^{-i\beta_l(j)z}$ on the diagonal, respectively, and, similarly for region $(j+1)$. N_l is a mode normalization factor given by $N_l = 1/\sqrt{2\beta_l}$. $t = (t_1, t_2, ..., t_N)$ and $r = (r_1, r_2, ..., r_N)$, where N is the number of ideal modes used to describe the solution in each region (Eq. (50)).

Using Eq. (54) the transfer matrix connecting regions j and $j+1$ can be calculated. If one connects the regions in the forward direction (i.e., from region j to $j+1$) one obtains

$$\begin{pmatrix} t^{(j+1)} \\ r^{(j+1)} \end{pmatrix} = \mathbf{M}^{(j)} \begin{pmatrix} t^{(j)} \\ r^{(j)} \end{pmatrix}, \tag{57}$$

where

$$\mathbf{M}^{(j)} = \left[\mathbf{U}^{(j+1)}\mathbf{E}^{(j+1)}\right]^{-1}\left[\mathbf{U}^{(j)}\mathbf{E}^{(j)}\right] \tag{58}$$

and the total 2Nx2N transfer matrix

$$\mathbf{M} = \begin{pmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} \\ \mathbf{M}_{21} & \mathbf{M}_{22} \end{pmatrix} \tag{59}$$

is given by

$$\mathbf{M} = \mathbf{M}^{(J)} ... \mathbf{M}^{(2)}\mathbf{M}^{(1)}, \tag{60}$$

where J is the total number of regions into which the area with z-dependent refractive index was divided. To calculate the reflection and transmission coefficients obtained in the regions before and after the area with z-dependent refractive index we use the transfer matrix from Eq. (60) and impose the correct boundary conditions which state that after the area with z-dependent refractive index there are only transmitted waves. Therefore,

$$\begin{pmatrix} T \\ 0 \end{pmatrix} = \begin{pmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} \\ \mathbf{M}_{21} & \mathbf{M}_{22} \end{pmatrix} \begin{pmatrix} C \\ R \end{pmatrix}, \tag{61}$$

where $C = (C_1, C_2, \ldots, C_N)$ are the coefficients of the incoming optical modes, $R = (R_1, R_2, \ldots, R_N)$ and $T = (T_1, T_2, \ldots, T_N)$ are the reflection and transmission coefficients of optical modes obtained as a result of the z-dependence of the refractive index. From Eq. (61) we obtain

$$R = -\mathbf{M}_{22}^{-1}\mathbf{M}_{21}C \tag{62}$$

and

$$T = [\mathbf{M}_{11} - \mathbf{M}_{12}\mathbf{M}_{22}^{-1}\mathbf{M}_{21}]C. \tag{63}$$

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