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Influence of root resistivity on plant water uptake mechanism, Part II: analytical solutions for low/moderate soil-root conductivity ratio

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Abstract A one-dimensional approximate analytical model, which preserves the main features of soil-crop-atmospheric hydrodynamics, has been suggested for plant roots of low soil-root conductivity ratio (SRCR). The proposed approach involves physically based concepts, such as mass balance equation, Darcy's law, and related water uptake and plant transpiration functions. Two main assumptions have been made to derive the analytical solution: (1) gravitational flow is adopted and (2) the uniform soil moisture distribution within the root water activity zone is supposed. The mass balance equation in its integral form is solved by the method of characteristics. This leads to the two functional equations for soil pressure head and root potential, which can be solved simultaneously by using common software. The model has been further verified against the numerical one. The model represents a reasonable compromise between the complicated mechanism of unsaturated water flow with root water uptake (RWU) and still insufficient knowledge of the soil-plant-atmospheric continuum. It is able to account for temporal fluctuations in root activity zone and provides a relatively simple algorithm for investigation of RWU-mechanism. Besides the theoretical and applicative importance, this flow model yields water and velocity distributions within soil profile, and, thereby, constitutes a preliminary step toward solution of contaminant transport problems in vadose zone.

Keywords Root water uptake · Soil-root conductivity ratio · Moving Uptake front · Approximate analytical model

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1 Introduction

The increasing demand and deteriorate quality of water, primarily, in arid and semi-arid areas of the world requires continual assessment of the management of this valuable resource (Cardon and Letey 1999), which in turn requires the design of an adequate model of spatial and temporal distribution of water within shallow soil layers. The flow of water in unsaturated soils involves highly nonlinear interactions between the capacity of soils for storing water, and the moisture transport due to suction as well as gravitational forces (Raats 1976). In the presence of plants, uptake patterns introduce additional complexity to modeling soil water dynamics (Mmolawa and Or 2001).

Many studies are devoted to multidimensional modeling of the processes related to soil-plant-atmospheric continuum. Today, developing of two- and even three-dimensional computer models becomes less and less limited by computer memory and time requirements (Russo et al. 1998). However, the current knowledge of the plant activity, such as root water and nutrient uptake, root growth, soil-plant-atmospheric interactions, is still insufficient to provide information necessary for comprehensive multidimensional modeling. On the other hand, strong nonlinearity of the flow equations in unsaturated zone eventually precludes attaining exact solutions.

Under such circumstances, simpler models, which take into account the main features of the water flow in vadose zone, on the one hand, and allow one to derive analytical or semi-analytical approximate solutions, on the other hand, are highly desirable. Establishing approximate analytical model of moisture transport with RWU can be a proper alternative to existing time-consuming computational procedures.

The main objective of this study is to develop such a model and apply it to investigation of water transport mechanism in upper soil layers with root uptake.

2 Modeling unsaturated vertical flow with root water uptake

Modeling soil-root water interaction requires solving the moisture transport equation in partially saturated soil with RWU introduced as a sink-term. The general form of one-dimensional flow equation results by adding the sink-function U that accounts for the vertical and temporal distribution of water extraction by roots, to Richards's equation:

$$\Delta \cdot \frac{\partial S(z,t)}{\partial t} - \frac{\partial}{\partial z}\left(K(S) \cdot \frac{\partial h(S)}{\partial z} - K(S)\right) = -U(z,t), \tag{1}$$

where $S = (\theta - \theta_r)/\Delta$ is the water saturation with $\Delta = \theta_s - \theta_r$ denoting the moisture capacity, θ representing the volumetric water content, and θ_s and θ_r being the saturated and irreducible water contents, respectively. In (1) $K(S) = K_\mathrm{s} \cdot K_\mathrm{p}(S)$ is the soil hydraulic conductivity with K_s and $K_\mathrm{p}(S)$ being the saturated and relative conductivities, respectively; $h(S)$ is the soil pressure head (taken here as positive), t is the time and z is the soil depth positively directed downward.

Often to avoid complete three-dimensional simulations, a *macroscopic scale* approach that disregards the flow patterns toward individual roots and represents the root system as a vertical sink is adopted (e.g. van den Honert 1948; Ogata et al. 1960; Gardner 1964; Rose and Stern 1967; Whisler et al. 1968; Nimah and Hanks, 1973a, b; Feddes et al. 1974; Rowse et al. 1978; Hillel 1980; Molz 1981; Bresler et al.

1982; Hupet et al. 2003; Šimunek et al. 2006). Following the analogy to Omh's or Darcy's law the moisture flux from soil to roots can be assumed to be proportional to the hydraulic conductance of the entire system (soil and root), and to the difference between total pressure head at the root-soil interface and corresponding pressure head of the soil. Neglecting the root resistance, the sink-term entering into (1) was first introduced by Gardner (1964):

$$U(z,t) = b(z) \cdot K(S) \cdot (h_r(t) - h(S)) \cdot H\left[h_r - h(z)\right], \tag{2}$$

where $b(z)$ is the root distribution function, $h_r(t)$ is the root water potential that alters with time but is assumed to be constant with depth, and $H\left[h_r(t) - h(S)\right]$ is the Heaviside function:

$$H[x] = \begin{cases} 0 & \text{for} \quad x < 0 \\ 1 & \text{for} \quad x \geq 0 \end{cases} \tag{3}$$

Nevertheless, the literature review presented in a preceding paper (Levin et al. 2007), shows that there is no clear evidence regarding the leading role of either soil or root resistances in the RWU-process. Some studies conclude (e.g. Newman 1969a, b; Gardner and Ehlig 1962; Taylor and Klepper 1975; Herkelrath et al. 1977; Jones et al. 1982; Zur et al. 1982; Lynn and Carlson 1990; Taylor et al. 1992) that the major resistance to water flow is associated with the plant, whereas soil resistance would become significant only at soil saturations near the wilting point (Zur et al. 1982). However, there is also experimental evidence in favor of much smaller root resistances (Reid and Huck 1990; Gardner 1991; Nobel and Cui 1992; Nobel 1999; Polak and Wallach 2001; Li et al. 2002a, b).

The sink-function in form (2) introduces an additional unknown, $h_r(t)$, to Eq. (1), hence requiring the specification of an additional condition. In the practical range of RWU, the transpiration rate $T(t)$ is entirely due to the plant transport and, hence, it can be determined by the RWU-rate

$$T(t) = \int_0^L U(z,t)dz, \tag{4}$$

where L is the rooting length. To account for reduction in transpiration rate resulted from water stress the actual transpiration rate is often related to the potential one by:

$$T(t) = f \cdot T_p, \tag{5}$$

where f is a reduction factor dependent on the soil moisture. Observations show that the transpiration rate is affected by the soil saturation only at small water contents near the wilting point (Gardner 1991), so that the water flux is primarily controlled by the atmospheric conditions (Novak 1986) and, for instance, can be determined by Penman-Monteinth's formula (Monteinth 1965) or by one of its modifications.

Assuming that as long as soil water content averaged over the root zone, θ_{av}, is larger than θ at field capacity, θ_{fc}, the plant transpiration rate is equal to the potential transpiration and at smaller θ it decreases linearly, one gets:

$$f = \begin{cases} 1 & \text{for} \quad \theta \geq \theta_{fc} \\ \frac{\theta_{av}-\theta_w}{\theta_{fc}-\theta_w} & \text{for} \quad \theta < \theta_{fc} \; \theta_{av} = \frac{1}{L} \cdot \int_0^L \theta(t,z)dz \end{cases} \tag{6}$$

where θ_w denotes the volumetric water content at wilting.

It should be emphasized that accounting for RWU leads to a complicated nonlinear flow equation with extremely small chances of deriving exact solutions. Few attempts to obtain analytical results were undertaken for simplified flow equations. Thus, Warrick (1974) and Lomen and Warrick (1976) solved the linearized one-dimensional steady-state flow equation for various sink-functions. Raats (1976) presented several analytical solutions for special cases of the flow equation with water extraction. Often to attain an analytical solution to flow problem and to simplify its mathematical derivation the RWU is assumed to be an exponential function of rooting depth (Rubin and Or 1993; Coelho and Or 1996; Basha 2000; Schoups and Hopmans 2002; Yuan and Lu 2005). However, due to the strong nonlinearity of the problem, most investigations apply various numerical and empirical models for analyzing dynamics of water propagation through the root zone (e.g. Nimah and Hanks 1973a, b; Feddes et al. 1976; Šimunek et al. 2006).

3 Analytical model for low/moderate soil-root conductivity ratio

Instead of giving prevalence to the soil conductivity or the root resistivity, the modified *Gardner's* sink-term model (2) is adopted with the RWU proportional to the total hydraulic conductivity $K_t(S)$:

$$U(z,t) = b(z) \cdot K_t(S) \cdot (h_r(t) - h(S)) \cdot H\left[h_r(t) - h(S)\right] \tag{7}$$

where $K_t^{-1} = K^{-1} + K_r^{-1}$ is the total hydraulic resistivity.

Although the root resistivity may slightly change with environmental conditions (Hillel 1980), it is fairly independent upon the transpiration (e.g. Lynn and Carlson 1990; Taylor et al. 1992; Humphries and Long 1995) and can be assumed constant over a large range of soil moisture. As mentioned in Levin et al. (2007), the SRCR has a significant influence on the mechanism of RWU. Unfortunately, this ratio is still not determined properly, i.e. the range of magnitudes is: $1 \leq K/K_r \leq 10^8$.

It is emphasized that Eq. (7) precludes an inverse flow from roots to soil and assumes a uniform distribution of the root water potential along the root depth (Gardner 1964). As it is seen from (7), the rate of RWU is state dependent in terms that the root extracting area, determined by depths, where $h(S) < h_r(t)$, is not known a priori. The location of this area changes in time and is determined by solving the flow (1) and plant water status (4)–(6) equations. Additionally, the plant potential $h_r(t)$ is bounded by the wilting pressure head h_w. Before the wilting occurs ($t < t_w$), the vertical distribution of U and $h_r(t)$, though satisfying the condition (5) is unknown a priori.

In Levin et al. (2007) a numerical code, based on the time-weighted finite difference scheme, has been developed for solving the flow equation (1) with a sink-term (7). The numerical model has been applied to simulate soil moisture flow during redistribution in initially uniformly wetted soil as well as to simulate water penetration during infiltration-redistribution cycles. Calculations show that for plants of relatively small root resistivity, the zone of the water extraction moves with time, exhibiting the so-called "Moving Uptake Front" (MUF) effect found in a few field experiments (Reid and Huck 1990; Polak and Wallach 2001; Li et al. 2002a, b). On the other hand, the mechanism of the water extraction is considerably simpler for plants of high root resistivity. For such crops, the MUF effect has not been obtained and roots extract water along the root length L proportionally to the local root density $b(z)$. These findings allow a significant simplification of the flow model. The approximate analytical solution presented in this paper is valid for low/moderate SRCR (i.e. $K_s/K_r \leq 10^4$) and is based on the assumptions coming from the numerical simulations.

The analysis of the water content profile obtained in the course of numerical simulations for low/moderate SRCR permits one to suggest an approximate analytical model of flow with root uptake for this case. The analytical model for low/moderate SRCR developed here follows the approach suggested recently for the unsaturated flow without roots (Lessoff and Indelman, 2004). The main assumptions are as follow:

1. *The gravitational flow is adopted*. Since the MUF generation is due to the drainage of the upper soil layers, it is believed that the cause of its appearance does not depend on the capillary force, although the latter may somehow affect the location of the MUF.
2. Analysis of numerical computations of the water content for $K_s/K_r = 10^3$ and for various model parameters presented in the preceding paper (Levin et al. 2007) has shown that *the θ-profile within the root uptake zone is practically uniform*. Note that this assumption is in an agreement with field experiments that usually show the nearly uniform water content in the root zone (e.g. Raats 1976).

3.1 Root uptake for redistribution in uniformly wetted soil

We consider vertical water penetration in a one-dimensional semi-infinite domain $z > 0$. The water redistribution takes place in the upper soil layer with an initial constant water content θ_0. The flow equation (1) is solved for the initial condition of constant saturation and boundary one of no-flux at the soil surface

$$S(z,0) = S_0 = \frac{\theta_0 - \theta_r}{\theta_s - \theta_r}, \quad K(S) \cdot \left(\frac{\partial h(S)}{\partial z} - 1 \right) \bigg|_{z=0} = 0 \tag{8}$$

Since at $t = 0$ the whole root extracts water, the initial root potential is obtained by combining (4)–(6) with (7) as follows

$$h_r(t=0) = h(S_0) + \frac{T}{L \cdot K_t(S_0) \cdot \overline{b}}, \tag{9}$$

where $\overline{b} = L^{-1} \cdot \int_0^L b(z)dz$ is the average root density.

The above-mentioned assumptions permit us to describe the water content profile by dividing the flow domain into three representative regions (Fig. 1a). In the upper soil layers (*Domain I*) no water uptake takes place. It is convenient to introduce a new (artificial) parameter, namely "root volumetric water content", θ_{tr}, that supposedly can be related to the root potential through the retention curve normally used for soil (e.g. see below (32)), i.e. $h_r(t) = h(\theta_{tr})$ or $h_r(t) = h(S_{tr})$. In this domain an expansion wave drains the soil in such a way that θ varies from θ_r to θ_{tr} while z grows from 0 to $z_{tr}(t)$, where the latter is the location of the MUF. The θ-profile within the *Domain I* is determined by the characteristic equation

$$\frac{dz}{dt} = \frac{1}{\Delta} \cdot \frac{dK(S)}{dS}, \quad S = const \tag{10}$$

Domain II corresponds to depths $z_{tr}(t) \leq z \leq L$ where the root takes up water. In this domain $\theta = \theta_u = const$ and the transpiration is determined by the substitution of (7) into (4) as follows

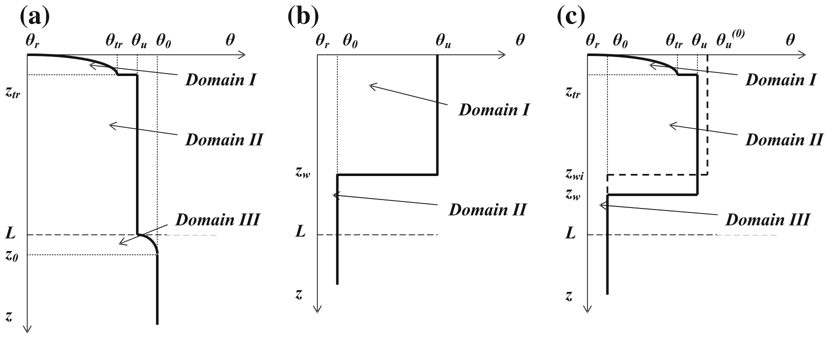

Fig. 1 Approximate soil moisture profile: during the redistribution in the initially uniformly wetted soil **(a)**; during the infiltration **(b)** and the following redistribution **(c)** for soil-root conductivity ratios $K_s/K_r \leq 10^4$

$$T(t) = \int_{z_{tr}(t)}^{L} U(z,t)dz = K_t(S_u) \cdot \left[h(S_{tr}) - h(S_u)\right] \cdot \int_{z_{tr}(t)}^{L} b(z)dz \tag{11}$$

Finally, *Domain III* corresponds to the free drainage below the root zone $z > L$. Here the water content varies from θ_u at $z = L$ to the initial value θ_0 at depth z_0 determined from Eq. (10), and remains constant below z_0.

These two assumptions allow us to write the mass balance equation from $z = 0$ to depth z_0 in a simple form

$$\Delta \cdot \int_0^{z_0} [S_0 - S(z,t)]\, dz = K(S_0) \cdot t + \int_0^t T(\tau)d\tau \tag{12}$$

The spatial integral in (12) can be calculated as follows

$$\int_0^{z_0} [S_0 - S(z,t)]\, dz = S_0 \cdot z_0 - \int_0^{z_{tr}(t)} S(z,t)dz - \int_{z_{tr}(t)}^{L} S(z,t)dz - \int_L^{z_0} S(z,t)dz \tag{13}$$

In the second integral on the r.h.s. of (13) $S(z,t) = S_u(t)$ does not depends on z. The first integral in (13) is computed with the aid of (10) as follows

$$\begin{aligned}\int_0^{z_{tr}} S(z,t)dz &= \int_0^{S_{tr}} [z_{tr} - z(S,t)]dS = z_{tr} \cdot S_{tr} - \int_0^{S_{tr}} \left[\frac{t}{\Delta} \cdot \frac{dK(S)}{dS}\right] dS \\ &= \frac{t}{\Delta} \cdot \left[S_{tr} \cdot \frac{dK(S_{tr})}{dS} - K(S_{tr})\right],\end{aligned} \tag{14}$$

whereas the rest of the terms in (13) are approximated by

$$\begin{aligned}S_0 \cdot z_0 - \int_L^{z_0} S(z,t)dz &= S_0 \cdot L - \int_L^{z_0} [S_0 - S(z,t)]\, dz \\ &= S_0 \cdot L - \int_{S_u}^{S_0} [z(S,t) - z(S_u,t)]\, dS \\ &= S_0 \cdot L - \frac{t}{\Delta} \cdot \left[K(S_0) - K(S_u) - (S_0 - S_u) \cdot \frac{dK(S_u)}{dS}\right]\end{aligned} \tag{15}$$

Finally, using (5) the integral in the r.h.s. of (12) is given by $\int_0^t T(\tau)d\tau = T_\mathrm{p} \cdot \int_0^t f d\tau$, where f is expressed in terms of the average saturation

$$S_\mathrm{av}(t) = \frac{1}{L} \cdot \int_0^L S(t,z)dz = S_\mathrm{u} - \frac{t \cdot K_\mathrm{s}}{L \cdot \Delta} \cdot \left[(S_\mathrm{u} - S_\mathrm{tr}) \cdot \frac{dK_\mathrm{p}(S_\mathrm{tr})}{dS} + K_\mathrm{p}(S_\mathrm{tr}) \right] \quad (16)$$

For details see Appendix A.

With the aid of (6) and (16) we introduce a new function:

$$\chi(t) = \begin{cases} 1 & \text{for} \quad t \le t_\mathrm{fc} \\ \dfrac{t_\mathrm{fc}}{t} + \displaystyle\int_{t_\mathrm{fc}}^t \frac{S_\mathrm{av}(\tau) - S_\mathrm{w}}{S_\mathrm{fc} - S_\mathrm{w}} d\tau & \text{for} \quad t > t_\mathrm{fc} \end{cases}, \quad (17)$$

where t_fc is the time when the average saturation reaches field capacity $S_\mathrm{av}(t_\mathrm{fc}) = S_\mathrm{fc}$.

Substituting (14), (15) and (16) into (13) and the result into (12) yields the final expression for the mass balance

$$S_0 - S_\mathrm{u} - \frac{t \cdot K_\mathrm{s}}{L \cdot \Delta} \cdot \left[(S_0 - S_\mathrm{u}) \cdot \frac{dK_\mathrm{p}(S_\mathrm{u})}{dS} - (S_\mathrm{u} - S_\mathrm{tr}) \cdot \frac{dK_\mathrm{p}(S_\mathrm{tr})}{dS} + K_\mathrm{p}(S_\mathrm{u}) - K_\mathrm{r}(S_\mathrm{tr}) \right]$$
$$= \frac{T_\mathrm{p} \cdot t}{L \cdot \Delta} \cdot \chi(t) \quad (18)$$

Equations (11), (18) and (17) comprise the approximate analytical model of gravitational flow within the root zone during the redistribution in uniformly wetted soil for low-moderate SRCR. It permits one to calculate the two saturation values $S_\mathrm{u}(t)$ and $S_\mathrm{tr}(t)$ as functions of time. The MUF is further determined using the characteristic equation:

$$z_\mathrm{tr}(t) = \frac{t}{\Delta} \cdot \frac{dK\left[S_\mathrm{tr}(t)\right]}{dS} \quad (19)$$

3.2 Water uptake during infiltration-redistribution cycles

The solutions presented below consider only some situations that seem to be representative for water management of agricultural crops. The analytical solutions are derived in a similar manner as in the previous case (Sect. 3.1.). As before, during the infiltration-redistribution cycles the same assumptions are made: gravitational flow and constant soil moisture inside the root activity zone are supposed.

The specified initial condition corresponds to uniform soil moisture within the root zone, $\theta(z \ge 0) = \theta_0 = const$. Water is applied during t_a hours at constant rate r, smaller than the soil hydraulic conductivity at saturation K_s, and then the soil is left to drain freely, i.e. the initial and boundary conditions are:

$$S(z,0) = S_0 = \frac{\theta_0 - \theta_\mathrm{r}}{\theta_\mathrm{s} - \theta_\mathrm{r}}, \quad K(S) \cdot \left(\frac{\partial h(S)}{\partial z} - 1 \right)\bigg|_{z=0} = \begin{cases} r & \text{for} \quad 0 \le t < t_\mathrm{a} \\ 0 & \text{for} \quad t_\mathrm{a} \le t \end{cases} \quad (20)$$

In order to simplify the solution of the problem, the following additional assumptions are made:

1. θ_0 *is low enough so as to neglect the initial water movement* ($K(S_0) \approx 0$).
2. The wetting front moisture is believed to be considerably larger than θ_0, thus, the *root activity domain is* the region *limited from below by the wetting front* $z_\mathrm{w}(t)$ (see Fig. 1b, c), and *from above by soil surface* $z = 0$ *for the infiltration stage* (Fig.1b) and *by* $z_\mathrm{tr}(t)$ *during the following redistribution stage* (Fig. 1c).

Infiltration Stage

During this period two representative regions are recognized (Fig.1b):

Domain I is the region behind the wetting front ($0 \le z \le z_w$), i.e. a zone of the highest soil saturation and, consequently, the zone of crop water withdrawal.

Domain II is the zone below z_w, i.e. a region of no moisture movement and no water extraction.

The transpiration rate is estimated by replacing the integral limits in (11):

$$T(t) = \int_0^{z_w} U(z,t)dz = K_t(S_u) \cdot \left[h(S_{tr}) - h(S_u)\right] \cdot \int_0^{z_w} b(z)dz \tag{21}$$

The water balance equation is:

$$(S_u - S_0) \cdot z_w \cdot \Delta = q \cdot t - \int_0^t T(\tau)d\tau, \tag{22}$$

where q is the infiltration rate. Since the water application rate r is assumed to be smaller than K_s, the infiltration rate q is equal to r.

As piston flow is assumed, the wetting front propagation is approximated by:

$$z_w(t) = \frac{t}{\Delta} \cdot \frac{K\left[S_u(t)\right]}{S_u - S_0} \tag{23}$$

The combination of (22) with (23) yields:

$$(S_u - S_0) \cdot \frac{K(S_u)}{S_u - S_0} = q - T_p \cdot \chi(t), \tag{24}$$

where the average saturation in this case is given by:

$$S_{av}(t) = S_0 + (S_u - S_0) \cdot \frac{z_w}{L} \tag{25}$$

Finally,

$$K(S_u) = q - T_p \cdot \chi(t) \tag{26}$$

Soil saturation S_u and the wetting front location z_f are determined from (26) and (23), respectively. After that, the root potential is calculated using (21) as follows:

$$h_r(t) = h(S_u) + \frac{T(t)}{K_t(S_u) \cdot \int_0^{z_w} b(z)dz} \tag{27}$$

Redistribution Stage

This section deals with the conditions frequently observed in agriculture when the wetting front has not succeeded to reach the bottom of the root zone during the infiltration stage, i.e. at the beginning of redistribution $z_w(t = 0) = z_{wi} < L$ (Fig.1c). For convenience, the redistribution process is assumed to start at $t = 0$ and the initial soil saturation $S_u^{(0)}$ behind the wetting front, z_{wi} at this time corresponds to that estimated from (26) at the end of the previous (infiltration) stage.

In this case three representative domains are recognized:

Domain I is the same as in Sect. 3.1 ($0 \le z \le z_{tr}$ and $\theta_r \le \theta \le \theta_{tr}$).

Domain II is the root activity zone similar to that in Sect. 3.1 with a provision of z_w to be the lower boundary instead of L. This means, that the RWU-domain is now limited by two time-dependent fronts: by the upper one MUF or $z_{tr}(t)$, and by the lower MUF or $z_w(t)$. Hence, the transpiration rate is determined by replacing the lower limit of the integral in (11):

$$T(t) = \int_{z_{tr}(t)}^{z_w(t)} U(z,t)dz = K_t(S_u) \cdot \left[h(S_{tr}) - h(S_u)\right] \cdot \int_{z_{tr}(t)}^{z_w(t)} b(z)dz \tag{28}$$

Domain III is determined in a same manner as *Domain II* during the infiltration.

The water balance equation is:

$$\left(S_u^{(0)} - S_0\right) \cdot z_{wi} - (S_u - S_0) \cdot z_w + S_u \cdot z_{tr} - \int_0^{z_{tr}} S dz = \frac{1}{\Delta} \cdot \int_0^t T(\tau) d\tau \tag{29}$$

Using the characteristic equation (21), solution (14) and Eq. (23) one gets:

$$\begin{aligned} &\frac{t \cdot K_s}{\Delta} \cdot \left[(S_u - S_{tr}) \cdot \frac{dK_p(S_{tr})}{dS} + K_p(S_{tr})\right] - (S_u - S_0) \cdot z_w \\ &= \frac{T_p \cdot t}{\Delta} \cdot \chi(t) - \left(S_u^{(0)} - S_0\right) \cdot z_{wi} \end{aligned} \tag{30}$$

The average saturation in this case is given by:

$$S_{av}(t) = S_0 + (S_u - S_0) \cdot \frac{z_w}{L} - \frac{K_s \cdot t}{L \cdot \Delta} \cdot \left[(S_u - S_{tr}) \cdot \frac{dK_p(S_{tr})}{dS} + K_p(S_{tr})\right] \tag{31}$$

For details see Appendix A.

Equations (28) and (30) allow one to approximate two time-dependent saturation values $S_u(t)$ and $S_{tr}(t)$. The locations of wetting front $z_w(t)$ and $z_{tr}(t)$ are determined using (22) and (19), respectively.

3.3 Model verification

Verification of the analytical model is carried out by comparison of the analytical solution with the numerical one obtained in the preceding paper (Levin et al, 2007). The van Genuchten (1980) retention model for the soil pressure head h and the Mualem (1976) model of the water relative conductivity K_p were adopted in the calculations, respectively

$$S = \left[1 + (\alpha \cdot h)^n\right]^{-m}, \quad K_p = S^\lambda \cdot \left[1 - \left(1 - S^{1/m}\right)^m\right]^2, \tag{32}$$

where $\alpha, n, m = 1 - 1/n$ are empirical constants affecting the shape of the curves and λ is a pore-connectivity parameter.

The simulations are conducted for the input data shown in Table 1. As it has been shown in Levin et al. (2007), although the depth of the MUF penetration is highly sensitive to soil and root properties, calculations for typical depth varying density distributions show that qualitatively, the mechanism of RWU remains the same as for $b(z) = const$. Therefore, hereafter the presented results refer to the uniform root density along the root depth.

The analytical results for the water redistribution within the initially uniformly wetted soil and infiltration-redistribution cycle are compared in Figs. 2 and 3 with the numerical solution, respectively.

Figure 2 shows fairly good agreement between θ-profiles of these two models. The discrepancy is observed close to the soil surface and in the vicinity of the root

Table 1 Characteristics of the soil selected for simulation

Soil type	Loamy sand
Residual water content, θ_r	0.045
Saturated water content, θ_s	0.45
Water content at field capacity, θ_{fc}	0.218
Initial water content, $\theta_0 = \theta(S_0 = 0.5)$	0.2475
Saturated conductivity, K_s	2.0 $^{cm}/_{hr}$
α	0.06 cm^{-1}
n	1.65
$m = 1 - 1/n$	0.3939
Pore-connectivity parameter, λ	0.65
Average root density, $\bar{b}$	1.0 cm^{-2}
Root depth, L	80 cm
Root water potential at wilting, h_w	15000 cm
Potential transpiration rate, T_p (value averaged over a day)	0.0125 $^{cm}/_{hr}$

zone bottom, where the capillary forces play a significant role. In turn, neglecting the suction gradient in Darcy's law, results in some disagreement between the analytically and the numerically calculated MUFs. During the infiltration-redistribution cycle Fig. 3 the root activity domain is considerably smaller than in the previous case and, thus, more sensitive to suction gradients. Soil water distribution profiles presented in Fig. 3 demonstrate a greater difference between analytical and numerical solutions. Nevertheless, the analytical model grasps the main features of the water absorption mechanism, namely, the appearance of the uptake front, which moves downward at the beginning and returns upward at later stages of the drainage. This implies that the assumptions used for the analytical model do not change the main mechanism of the RWU. That is, the analytical solution preserves the main features of the MUF, i.e. its existence and the dependence of its location on the retention curve of the soil. Note also that the computer time required by the numerical simulation is about three orders of magnitude higher as compared with that needed for the analytical model.

4 Summary and conclusions

A simplified analytical model of moisture movement toward the roots has been developed for low/moderate SRCR (for the specified soil $K_s/K_r \leq 10^4$). Contrary to the previously suggested models, our approach takes into account both the soil conductivity and the root resistivity. The approximate solutions to the water flow problem have been derived to describe the moisture distribution and the RWU-patterns for rooted, homogeneous semi-infinite soil profile. At the upper boundary a surface influx is specified. Two major assumptions have been made to attain the analytical solutions: (1) the flux is gravitational; (2) the soil saturation along the root activity zone is uniform.

Two scenarios have been considered: (a) water penetration through initially uniformly wetted soil profile; (b) moisture movement during an infiltration-redistribution cycle (approximate solutions for other more complicated situations can be derived in the same manner). The comparison of the analytical results with the numerical solutions obtained in the previous study (Levin et al. 2007) shows that they are in

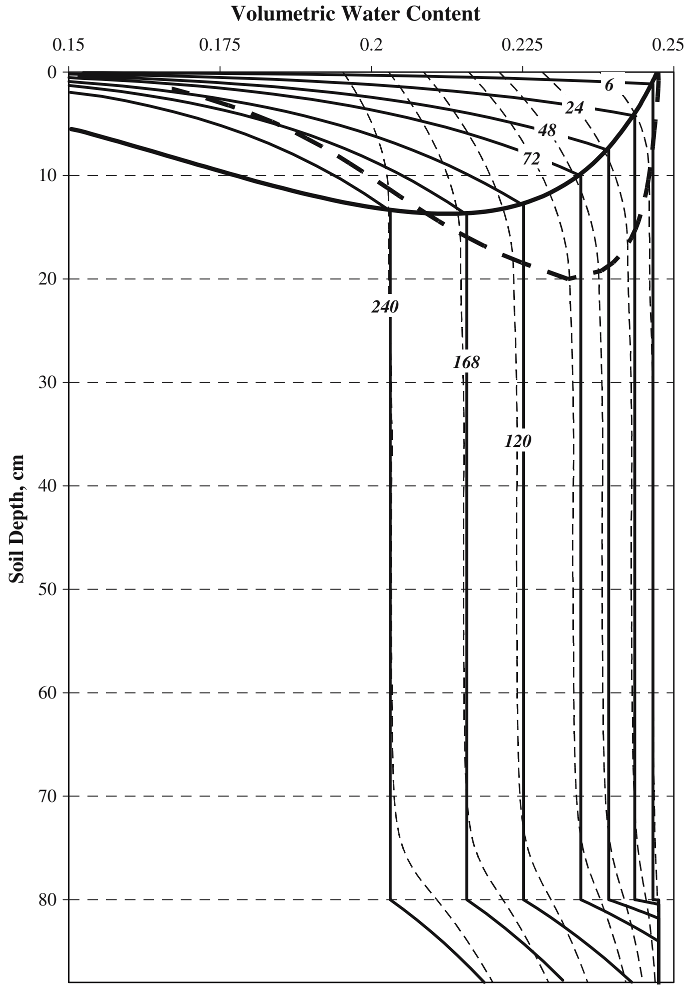

Fig. 2 Analytically calculated water content and MUF (solid lines) against numerical ones (dashed lines)

good agreement for the first scenario. Although the analytical model is not able to match exactly the numerical solution, primarily in those soil layers where the capillary forces are important, it gives an approximate distribution of soil moisture within the soil profile.

The model is physically based and preserves the main features of water flow and RWU processes. Thus, it represents a reasonable compromise between the complicated mechanism of unsaturated flow with RWU and our insufficient knowledge of plant water hydrodynamics. Although the solutions are restricted to the cases of low/moderate SRCR, they are easy to implement compared to numerical schemes and can be used for solving various problems encountered in soil water applications. Furthermore, the relatively simple structure of the analytical solutions permits one to treat problems associated with solute transport in the vadose zone.

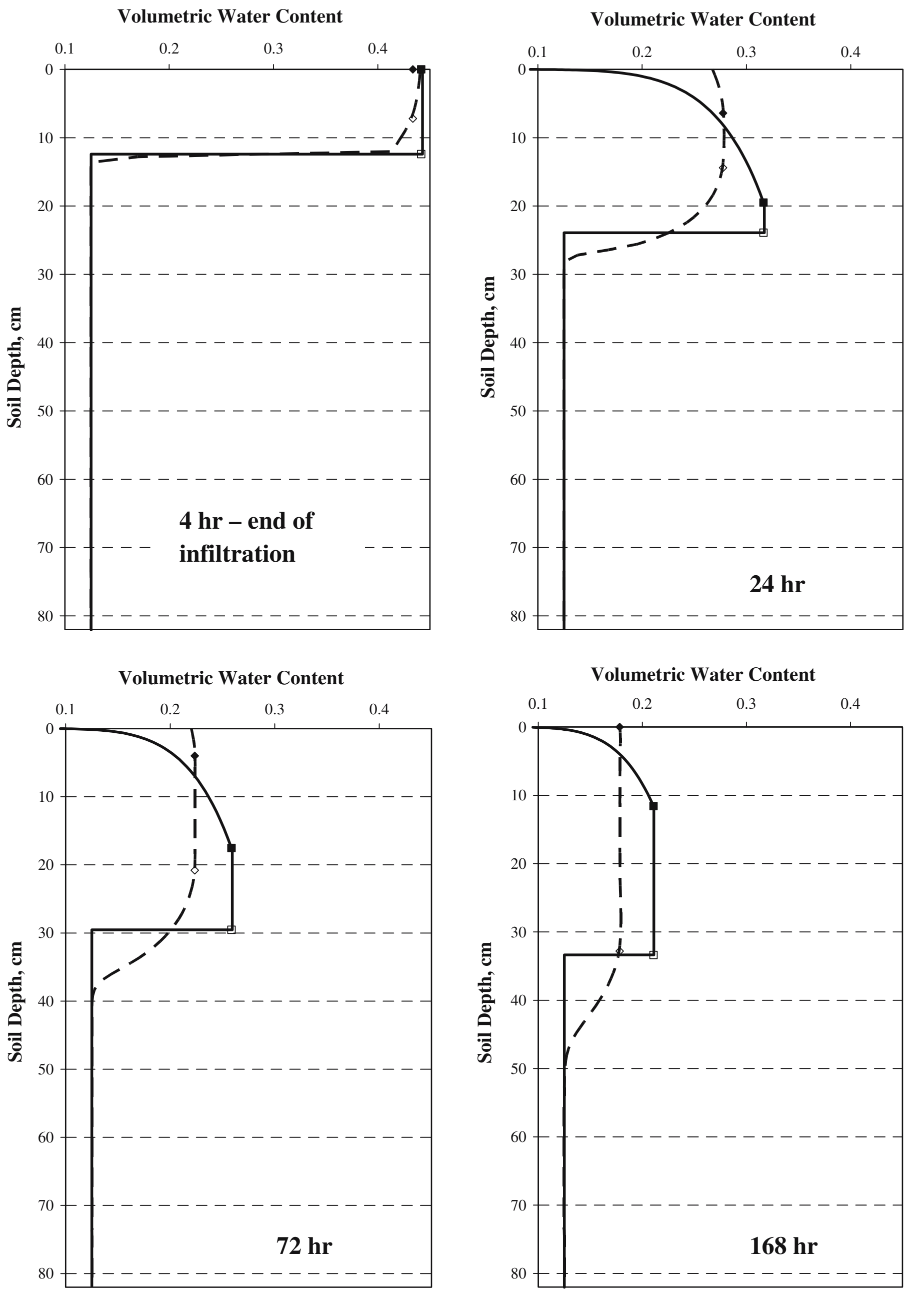

Fig. 3 Analytically calculated water content (solid lines) against numerical ones (dashed lines). Filled and empty squares illustrate the upper and lower MUF location, respectively

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Appendix A

The value of the soil saturation averaged over the rooting depth in (16), Sect. 3.1 is determined as follows:

$$S_{\mathrm{av}}(t) = \frac{1}{L} \cdot \int_0^L S(z,t)dz = \frac{1}{L} \cdot \left[\int^{Z_{\mathrm{tr}}} S(z,t)dz + S_{\mathrm{u}} \cdot (L - z_{\mathrm{tr}}) \right]$$

Using (14):

$$S_{\mathrm{av}}(t) = \frac{1}{L} S_{\mathrm{u}} + \frac{1}{L} \cdot \left[\frac{t}{\Delta} \left(\frac{dK(S_{\mathrm{tr}})}{dS} \cdot S_{\mathrm{tr}} - K(S_{\mathrm{tr}}) \right) - S_{\mathrm{u}} \cdot z_{\mathrm{tr}} \right]$$

Applying (19) and rearranging variables one gets:

$$S_{\mathrm{av}}(t) = S_{\mathrm{u}} - \frac{K_{\mathrm{s}} \cdot t}{L \cdot \Delta} \cdot \left[\frac{dK_{\mathrm{p}}(S_{\mathrm{tr}})}{dS} \cdot (S_{\mathrm{u}} - S_{\mathrm{tr}}) + K_{\mathrm{p}}(S_{\mathrm{tr}}) \right]$$

In order to obtain the average saturation in (31), Sect. 3.2, the same technique is utilized:

$$\begin{aligned} S_{\mathrm{av}}(t) &= \frac{1}{L} \cdot \int_0^L S(z,t)dz = \frac{1}{L} \cdot \left[\int_0^{z_{\mathrm{tr}}} Sdz + S_{\mathrm{u}} \cdot (z_{\mathrm{f}} - z_{\mathrm{tr}}) + S_0 \cdot (L - z_{\mathrm{f}}) \right] \\ &= \frac{1}{L} \cdot \left\{ \frac{t}{\Delta} \cdot \left[S_{\mathrm{tr}} \cdot \frac{dK(S_{\mathrm{tr}})}{dS} - K(S_{\mathrm{tr}}) \right] + (S_{\mathrm{u}} - S_0) \cdot z_{\mathrm{f}} - S_{\mathrm{u}} \cdot z_{\mathrm{tr}} + S_0 \cdot L \right\} \\ &= S_0 + (S_{\mathrm{u}} - S_0) \cdot \frac{z_{\mathrm{f}}}{L} - \frac{K_{\mathrm{s}} \cdot t}{L \cdot \Delta} \cdot \left[(S_{\mathrm{u}} - S_{\mathrm{tr}}) \cdot \frac{dK_{\mathrm{p}}(S_{\mathrm{tr}})}{dS} + K_{\mathrm{p}}(S_{\mathrm{tr}}) \right] \end{aligned}$$

References

Basha, H.A.: Multidimensional linearized nonsteady infiltration toward a shallow water table. Water Resour. Res. **36**, 2567–2573 (2000)

Bresler, E., McNeal, B.L., Carter, D.L.: Saline and sodic soils. Principles-dynamics-modeling. Springer-Verlag, Berlin Heidelberg New York (1982)

Cardon, G.E., Letey, J.: Plant water uptake terms evaluated for soil water and solute movement models. Soil Sci. Soc. Am. J. **32**, 1876–1880 (1992)

Coelho, E.F., Or, D.: A parametric model for two-dimensional water uptake intensity. Soil Sci. Soc. Am. J. **60**, 1039–1049 (1996)

Feddes, R.A., Bresler, E., Neuman, S.P.: Field test of a modified numerical model for water uptake by root systems. Water Resour. Res. **10**, 1199–1206 (1974)

Feddes, R.A., Kowalik, P.J., Malinka, K.K., Zaradny, H.: Simulation of field water uptake by plants using a soil water dependent root extraction function. J. Hydrology **31**, 13–26 (1976)

Gardner, W.R., Ehlig, C.F.: Some observations on the movement of water to plant roots. Agron. J. **54**, 453–456 (1962)

Gardner, W.R.: Relation of root distribution to water uptake and availability. Agron. J. **56**, 41–45 (1964)

Gardner, W.R.: Modeling water uptake by roots. Irrigation Sci. **12**, 109–114 (1991)

Herkelrath, W.N., Miller, E.E., Gardner, W.R.: Water uptake by plants. 2. The root contact model. Soil Sci. Soc. Am. J. **41**, 1039–1043 (1977)

Hillel, D.: Application of soil physics. Academic Press, New York (1980)

Humphries, S.W., Long, S.P.: WIMOVAC: a software package for modeling the dynamics of plant leaf and canopy photosynthesis. CABIOS **11**, 361–371 (1995)

Hupet, F., Lambot, S., Feddes, R.A., van Dam, J.C., Vanclooster, M.: Estimation of root water uptake parameters by inverse modeling with soil water content data. Water Resour. Res. **39**(11), 1312–1339 (2003)

Jones, J.W., Zur, B., Boote, K.J., Hammond, L.C.: Plant resistance to water flow in field soybeans: I. Non-limiting soil moisture. Agron. J. **74**, 92–98 (1982)

Lessoff, S.C., Indelman, P.: Analytical model of solute transport by unsteady unsaturated gravitational infiltration. J. Contaminant Hydrol. **72**, 85–107 (2004)

Levin A., Shaviv A., Indelman P.: Influence of root resistivity on plant water uptake mechanism, Part I: numerical solution. Transport Porous. Med. (2007) doi: 10.1007/s11242-006-9084-1

Li, Y., Fuchs, M., Cohen, Y., Wallach, R.: Water uptake profile response of corn to soil moisture depletion. Plant Cell Environ. **25**, 491–500 (2002a)

Li, Y., Wallach, R., Cohen, Y.: The role of soil hydraulic conductivity on the spatial and temporal variation of root water uptake in drip-irrigated corn. Plant Soil **243**, 131–142 (2002b)

Lomen, D.O., Warrick, A.W.: Solution of the one-dimensional linear moisture flow equation with implicit water extraction functions. Soil Sci. Soc. Am J. **40**, 342–344 (1976)

Lynn, B.H., Carlson, T.N.: A stomatal resistance model illustrating plant vs. external control of transpiration. Agr. Forest Meteorol. **52**, 5–43 (1990)

Mmolawa, K., Or, D.: Water and solute dynamics under a drip-irrigated crop: experiments and analytical model. Trans. ASAE **43**(6), 1597–1608 (2001)

Molz, F.J.: Models of water transport in the soil-plant system: a review. Water Resour. Res. **17**(5), 1245–1260 (1981)

Monteinth, J.L.: Evaporation and environment. Symp. Soc. Exp. Biol. **19**, 205–234 (1965)

Mualem, Y.: A new model for predicting the hydraulic conductivity of unsaturated porous media. Water Resour. Res. **12**, 513–552 (1976)

Newman, E.I.: Resistance to water flow in soil and plant: I. Soil resistance in relation to amount of root: theoretical estimates. J. Appl. Ecol. **6**, 1–2 (1969a)

Newman, E.I.: Resistance to water flow in soil and plant: II. A review of experimental evidence on the rhizosphere resistance. J. Appl. Ecol. **6**, 261–272 (1969b)

Nimah, M.N., Hanks, R.J.: Model for estimating soil water, plant and atmospheric interrelations, 1. Description and sensitivity. Soil Sci. Soc. Am. Proc. **37**, 522–527 (1973a)

Nimah, M.N., Hanks, R.J.: Model for estimating soil water, plant and atmospheric interrelations, 2. Field test for model. Soil Sci. Soc. Am. Proc. **37**, 528–531 (1973b)

Nobel, P.S., Cui, M.: Hydraulic conductances of the soil, the root-soil area gap, and the root: changes for desert succulents in drying soil. J. Exp. Bot. **43**, 319–326 (1992)

Nobel, P.S.: Physiochemical and Environmental Plant Physiology. Academic Press (1999)

Ogata, G., Richards, L.A., Fungardi, A.A., Drake, R.L.: Soil moisture availability for transpiration. Water Resour. Res. **4**(6), 1161–1169 (1960)

Polak, A., Wallach, R.: Analysis of soil moisture variations in an irrigated orchard root zone. Plant Soil **233**, 145–159 (2001)

Raats, P.A.C.: Analytical solutions of a simplified flow equation. Trans. ASAE **19**, 683–689 (1976)

Reid, J.B., Huck, M.G.: Diurnal variation of crop hydraulic resistance: A new analysis. Agron. J. **82**, 827–834 (1990)

Rose, C.W., Stern, W.R.: Determination of withdrawal of water from soil by crop roots as a function of depth and time. Aust. J. Res. **5**, 11–19 (1967)

Rowse, H.R., Stone, D.A., Gerwitz, A.: Simulation of the water distribution in soil. 2. The model for cropped soil and its comparison with experiment. Plant Soil **49**, 534–550 (1978)

Rubin, Y., Or, D.: Stochastic modeling of unsaturated flow in heterogeneous soils with water uptake by plant roots: The parallel columns model. Water Resour. Res. **29**, 619–631 (1993)

Russo, D., Zaidel, J., Laufer, A.: Numerical analysis of flow and transport in a three-dimensional partially saturated heterogeneous soil. Water Resour. Res. **34**(6), 1451–1468 (1998)

Schoups, G., Hopmans, J.W.: Analytical model for vadose zone solute transport with root water and solute uptake, available at www.vadosezonejournal.org., Vadose Zone J. **1**: 158–171 (2002)

Šimunek, J., Šejna, M., van Genuchten, M. Th.: The HYDRUS software package for simulating the two- and three-dimensional movement of water, heat, and multiple solutes in variably-saturated media, user manual, version 1.0, PC-Progress, Prague, Czech Republic (2006)

Taylor, H.M., Klepper, B.: Water uptake by cotton root systems: An examination of assumptions in the single root model. Soil Sci. **120**, 57–67 (1975)

Taylor, H.M., Upchurch, D.R., McMichael, B.L.: Root hydraulic resistance: Implication in modeling nutrient and water uptake. J. Plant Nutrition **15**, 727–736 (1992)
van den Honert, T.H.: Water transport as a catenary process. Faraday Soc. Discuss. **3**, 146–153 (1948)
van Genuchten, M.Th.: A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. Soil Sci. Soc. Am. J. **44**, 892–898 (1980)
Warrick, A.W.: Solution of the one-dimensional linear moisture flow equation with water extraction. Soil Sci. Soc. Am. Proc. **38**, 573–576 (1974)
Whisler, F.D., Klute, A., Millington, R.J.: Analysis of steady-state evapotranspiration from a soil column. Soil Sci. Soc. Am. Proc. **32**, 167–174 (1968)
Yuan, F., Lu, Z.: Analytical solutions for vertical flow in unsaturated, rooted soils with variable surface fluxes. Vadose Zone J. **4**, 1210–1218 (2005)
Zur, B., Jones, J.W., Boote, K.J., Hammond, L.C.: Total resistance to water flow in field soybeans: II. Limiting soil moisture. Agron. J. **74**, 99–105 (1982)